CENTROIDS
AND
CENTER OF GRAVITIES

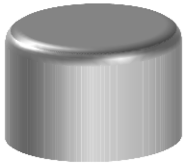
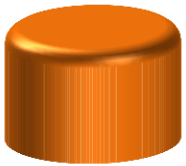
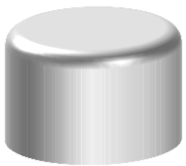
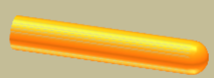
Mathematics possesses not only truth,
but supreme beauty - a beauty cold
and austere, like that of a sculpture.
Bertrand Russell

CONTENTS

Basic Definitions
Composite Objects

Definitions

Center of Gravity
Center of Mass
Volume Centroid
Area Centroid
Curve (line) Centroid

	Weights	W		Center of gravity
	Masses	$W=gM$	g gravitational acceleration	Center of mass
	Volumes	$W=g\rho V$	ρ density	Volume centroid
	Areas	$W=g\rho tA$	t thin plate thickness	area centroid
	Lines	$W=g\rho aL$	a wire cross section	line centroid

In our work we will consider the following

Uniform gravitational field.

density is uniform

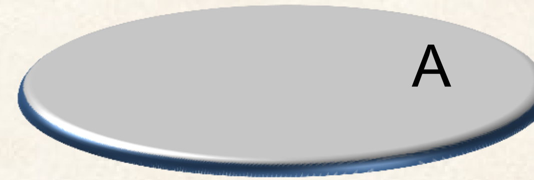
Uniform cross section

Thickness of thin plates is uniform

Area Centroid

given

$$x_G = \frac{\int x_e dW}{\int dW} \qquad y_G = \frac{\int y_e dW}{\int dW} \qquad z_G = \frac{\int z_e dW}{\int dW}$$



setting $dW = d(gm) = d(g\rho V) = d(\rho t A)$ leads to

$$x_G = \frac{\int x_e d(g\rho t A)}{\int d(g\rho t A)} \qquad y_G = \frac{\int y_e d(g\rho t A)}{\int d(g\rho t A)} \qquad z_G = \frac{\int z_e d(g\rho t A)}{\int d(g\rho t A)}$$

now, let us take g, ρ, and the body thickness t to be uniform and very small all over the body, then

$$x_G = \frac{\int x_e dA}{\int dA} \qquad y_G = \frac{\int y_e dA}{\int dA} \qquad z_G = \frac{\int z_e dA}{\int dA}$$

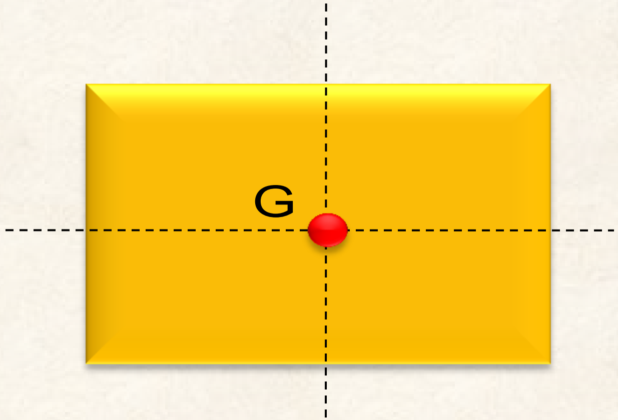
where dA Element of area

$A = \int dA$ Total area

Thus The Area Centroid Coincides With Its Center of Gravity

NOTICE

For uniform objects, if there are two or more lines of SYMMETRY, then the center of gravity is located on its intersection



Line Centroid

given

$$x_G = \frac{\int x_e dW}{\int dW} \qquad y_G = \frac{\int y_e dW}{\int dW} \qquad z_G = \frac{\int z_e dW}{\int dW}$$

setting $dW = d(gm) = d(\rho V) = d(\rho a L)$ leads to

$$x_G = \frac{\int x_e d(g\rho a L)}{\int d(g\rho a L)} \qquad y_G = \frac{\int y_e d(g\rho a L)}{\int d(g\rho a L)} \qquad z_G = \frac{\int z_e d(g\rho a L)}{\int d(g\rho a L)}$$

now, let us take g, ρ, and the wire cross section a to be uniform all over the body, then

$$x_G = \frac{\int x_e dL}{\int dL} \qquad y_G = \frac{\int y_e dL}{\int dL} \qquad z_G = \frac{\int z_e dL}{\int dL}$$

where dL Element of length

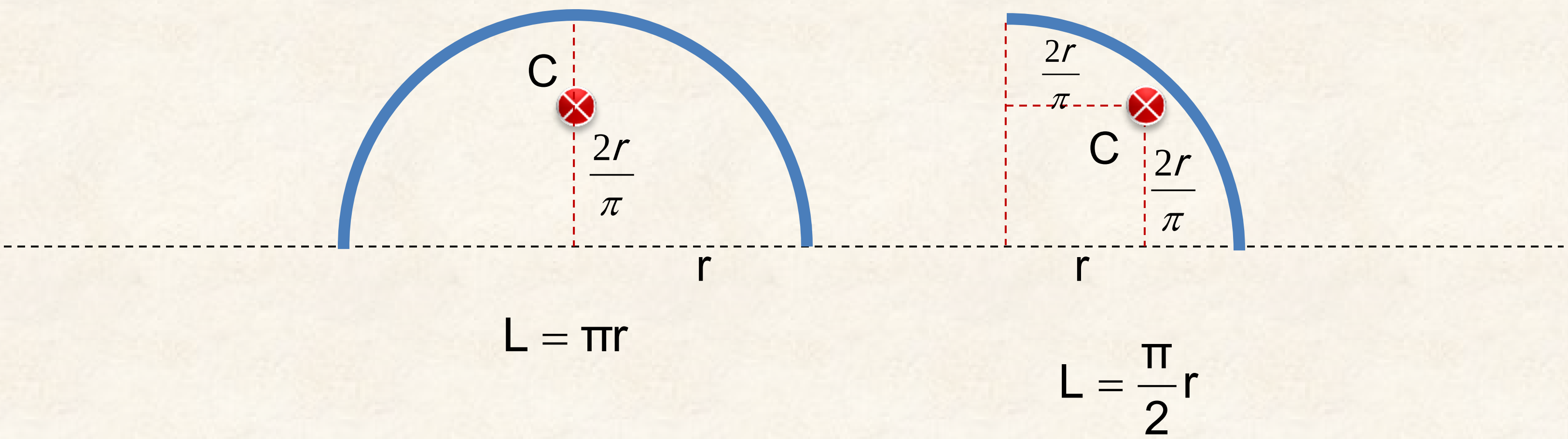
$L = \int dL$ Total length

Thus The Line Centroid Coincides With Its Center of Gravity

Composite Wires

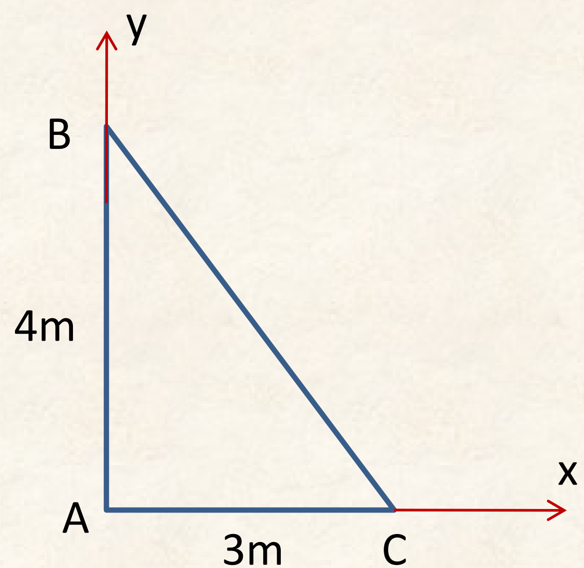
$$x_G = \frac{\sum_{i=1}^{n} x_i L_i}{\sum_{i=1}^{n} L_i} \qquad y_G = \frac{\sum_{i=1}^{n} y_i L_i}{\sum_{i=1}^{n} L_i} \qquad z_G = \frac{\sum_{i=1}^{n} z_i L_i}{\sum_{i=1}^{n} L_i}$$

Common Shapes and their centroids



Example-1

Determine the centroid of the three uniform wires AB, BC, and CA with respect to the x-y coordinates. All three wires have the same density and cross section



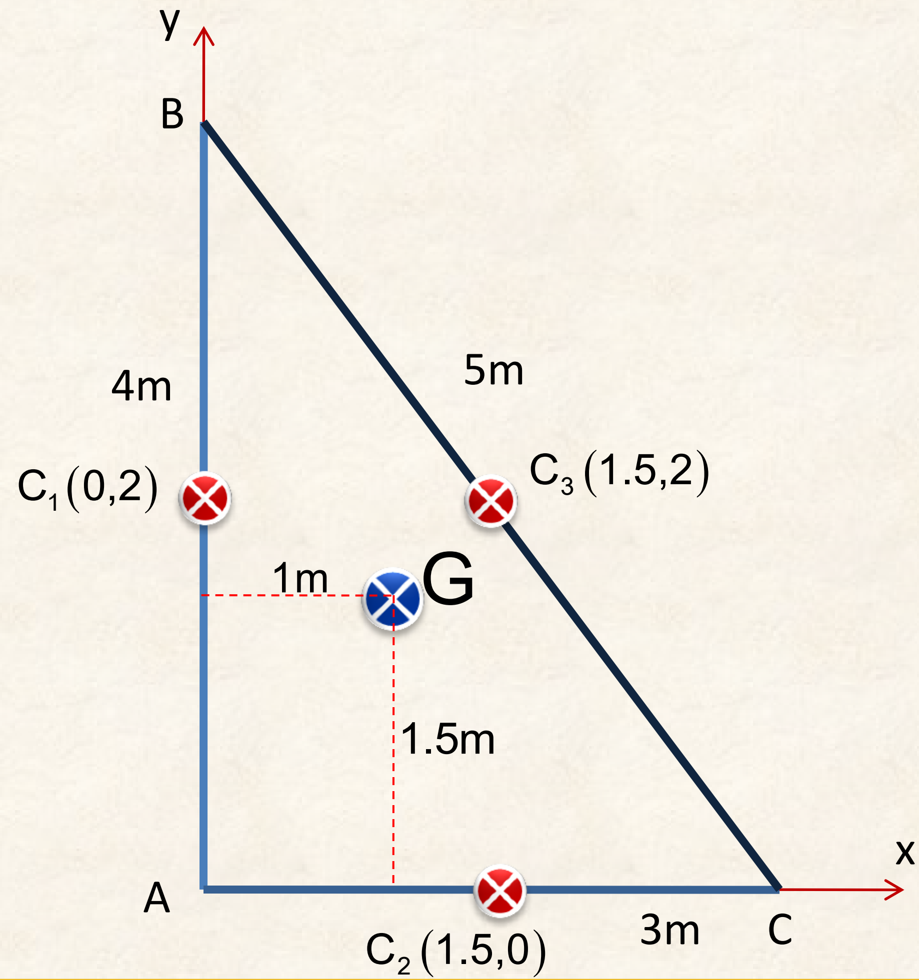
Solution

$$x_G = \frac{\sum_{i=1}^{n} x_i L_i}{\sum_{i=1}^{n} L_i}$$

$$x_G = \frac{4(0) + 3(1.5) + 5(1.5)}{4+3+5} = 1m$$

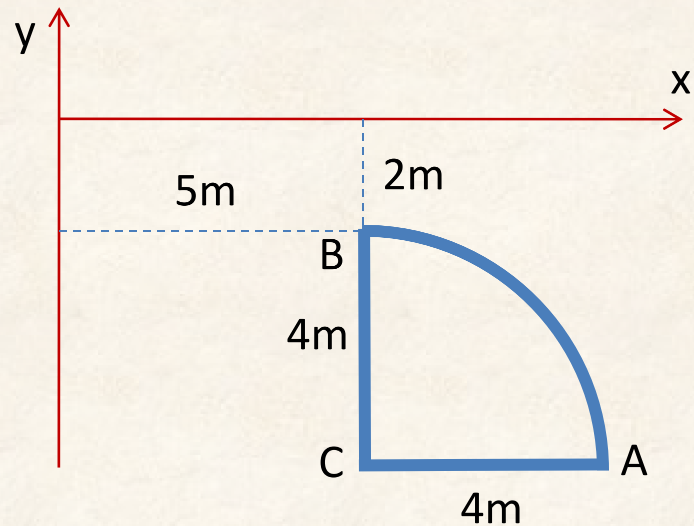
$$y_G = \frac{\sum_{i=1}^{n} y_i L_i}{\sum_{i=1}^{n} L_i}$$

$$y_G = \frac{4(2) + 3(0) + 5(2)}{4+3+5} = 1.5m$$

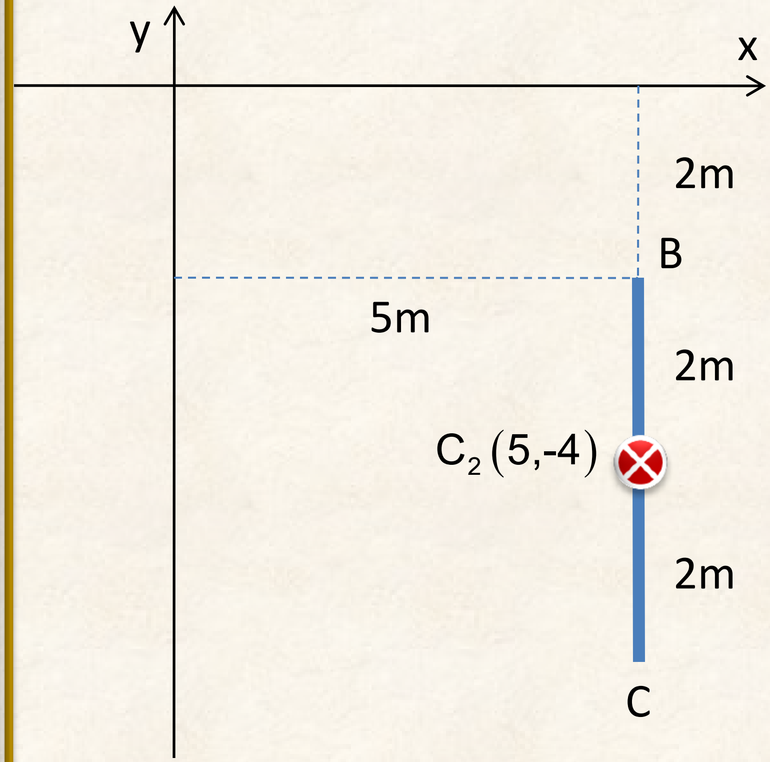


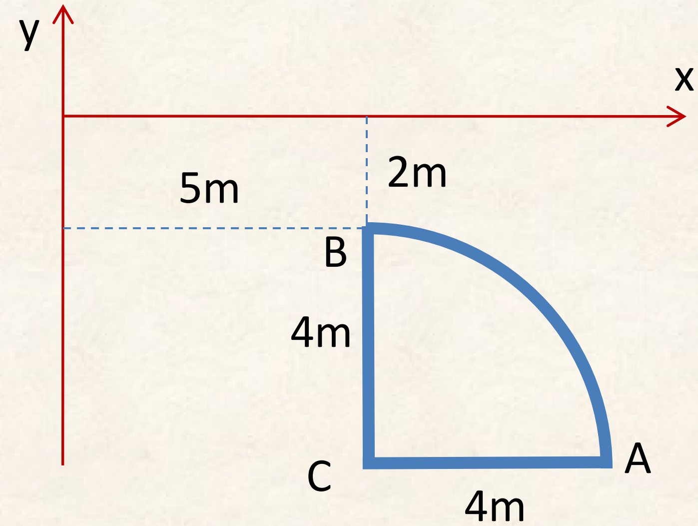
Example-2

Determine the centroid of the complete quarter circle shown with respect to the x-y coordinates. All wires have the same density and cross section

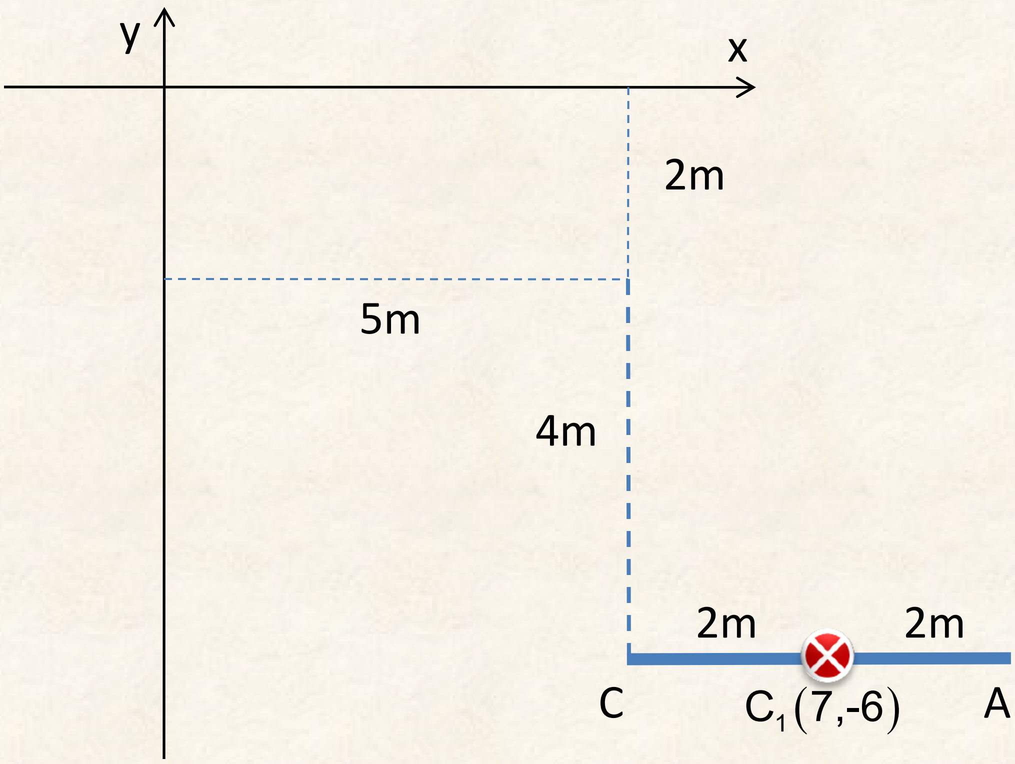


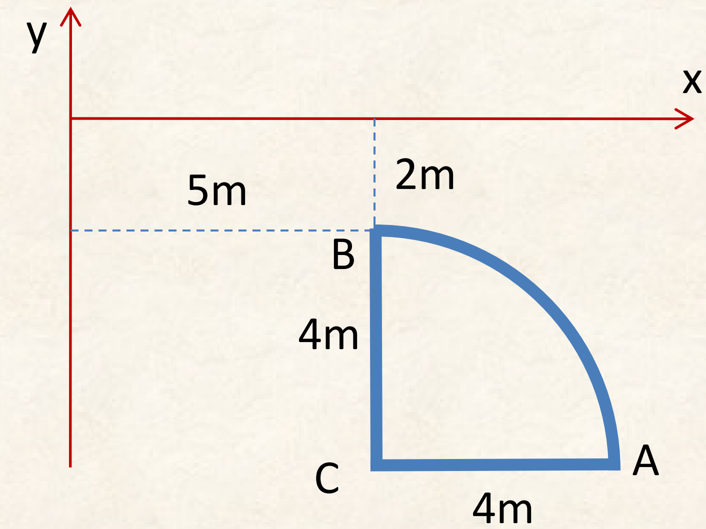
Solution

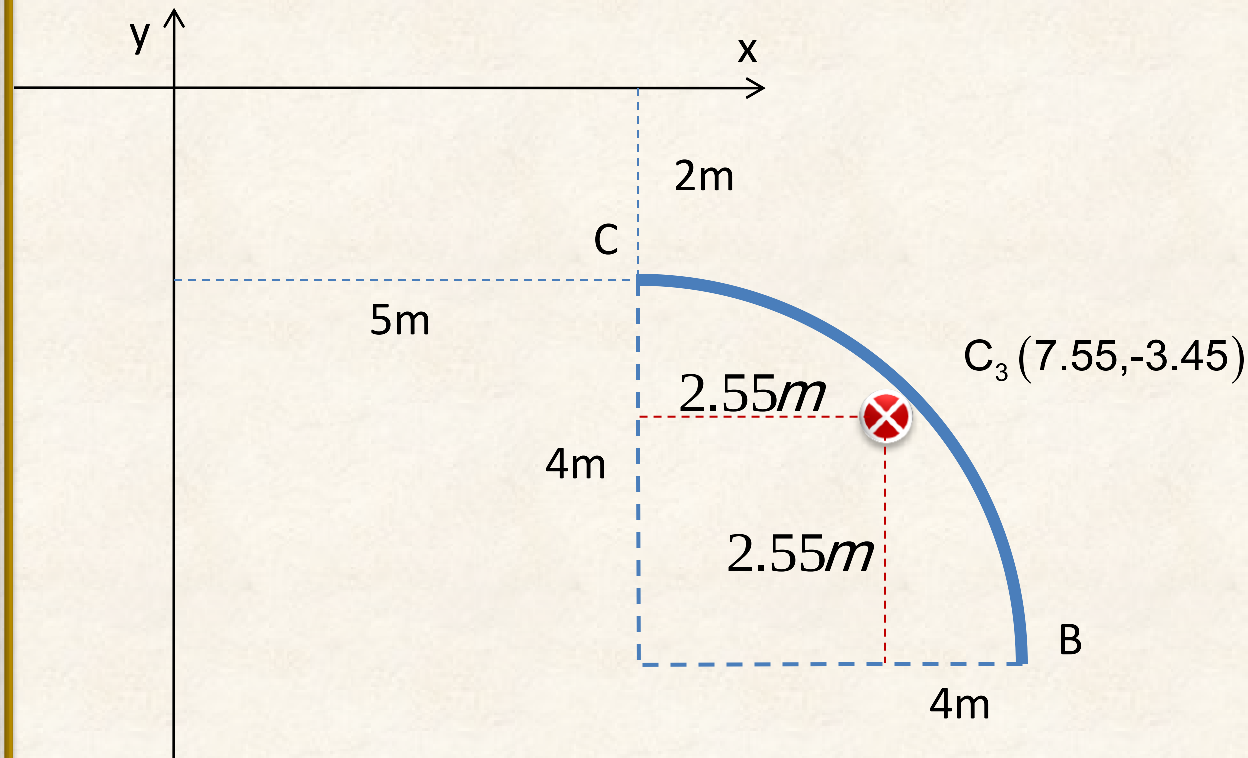


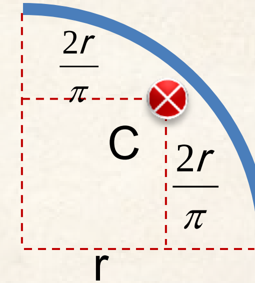


Solution









$$L = \frac{\pi}{2} r = 6.3$$

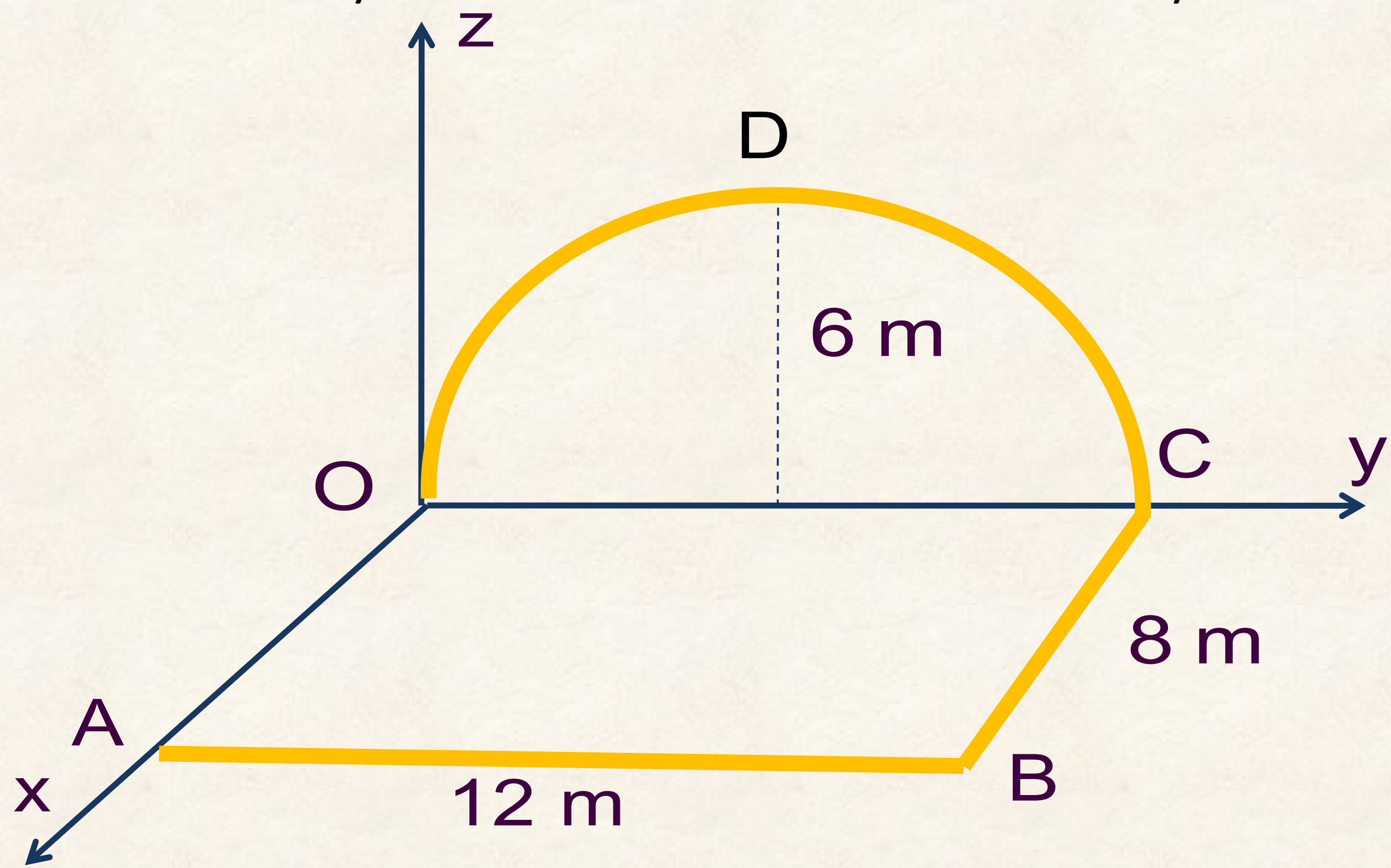
#	L	x	Lx	y	Ly
1	4	5	20	-4	-16
2	4	7	28	-6	-24
3	6.3	7.55	47.6	-3.45	-21.74
SUMS	14.3		95.6		-61.74

$$x_G = \frac{\sum_{i=1}^{n} x_i L_i}{\sum_{i=1}^{n} L_i} \qquad y_G = \frac{\sum_{i=1}^{n} y_i L_i}{\sum_{i=1}^{n} L_i}$$

$$x_G = \frac{95.6}{14.3} = 6.68m \qquad y_G = \frac{-61.74}{14.3} = -4.32m$$

EXAMPLE-3

Determine the centroid of the body shown. The wire is of uniform density and cross section



SOLUTION

Divide the wire into known sections

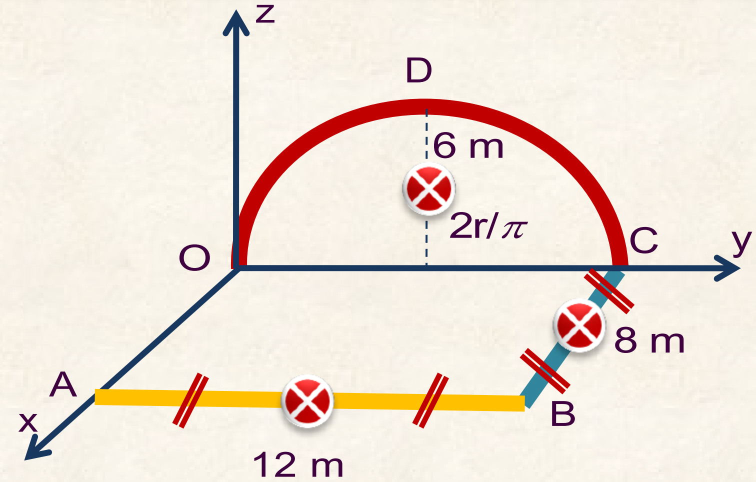
1- wire AB

2- wire BC

3- wire CDO

$$L_3 = CDO = \pi r = 3.14(6) = 18.85\,m$$

$$z_3 = \frac{2r}{\pi} = \frac{12}{3.14} = 3.82\ m$$



#	L	x	Lx	y	Ly	z	Lz
1	12	8		6		0	
2	8	4		12		0	
3	18.85	0		6		3.82	
SUMS							

3- The centroid of the whole wire is then located using;

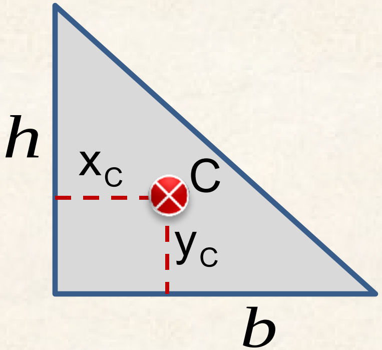
#	L	x	Lx	y	Ly	z	Lz
1	12	8	96	6	72	0	0
2	8	4	32	12	96	0	0
3	18.85	0	0	6	113.1	3.82	72
SUMS	38.85		128		281.1		72

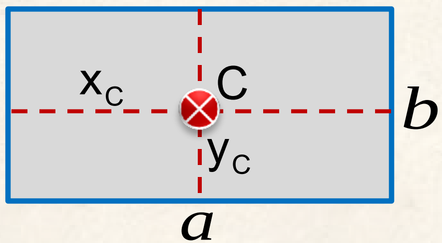
$$x_C = \frac{\sum_{i=1}^{n} L_i x_i}{\sum_{i=1}^{n} L_i} = \frac{128}{38.85} = 3.3 \; m$$

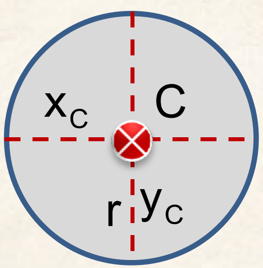
$$y_C = \frac{\sum_{i=1}^{n} L_i y_i}{\sum_{i=1}^{n} L_i} = \frac{281.1}{38.85} = 7.23 \; m$$

$$z_C = \frac{\sum_{i=1}^{n} L_i z_i}{\sum_{i=1}^{n} L_i} = \frac{72}{38.85} = 1.85 \; m$$

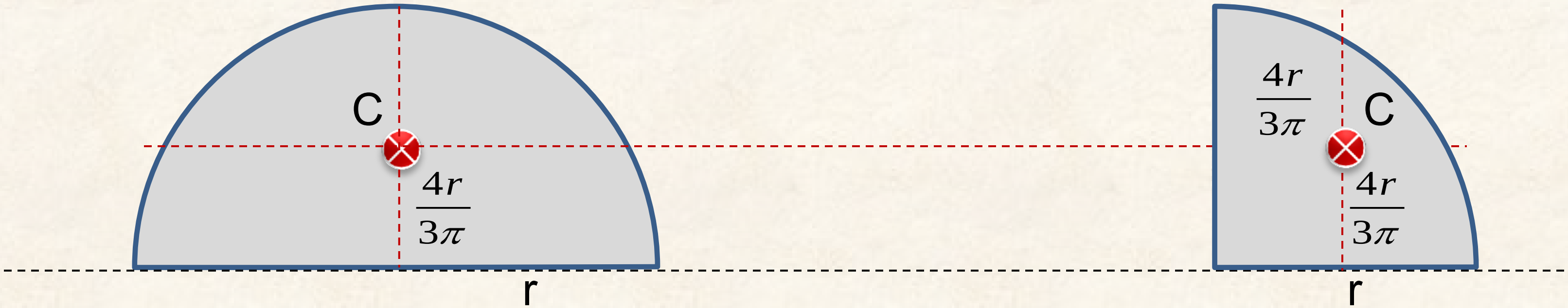
AREAS







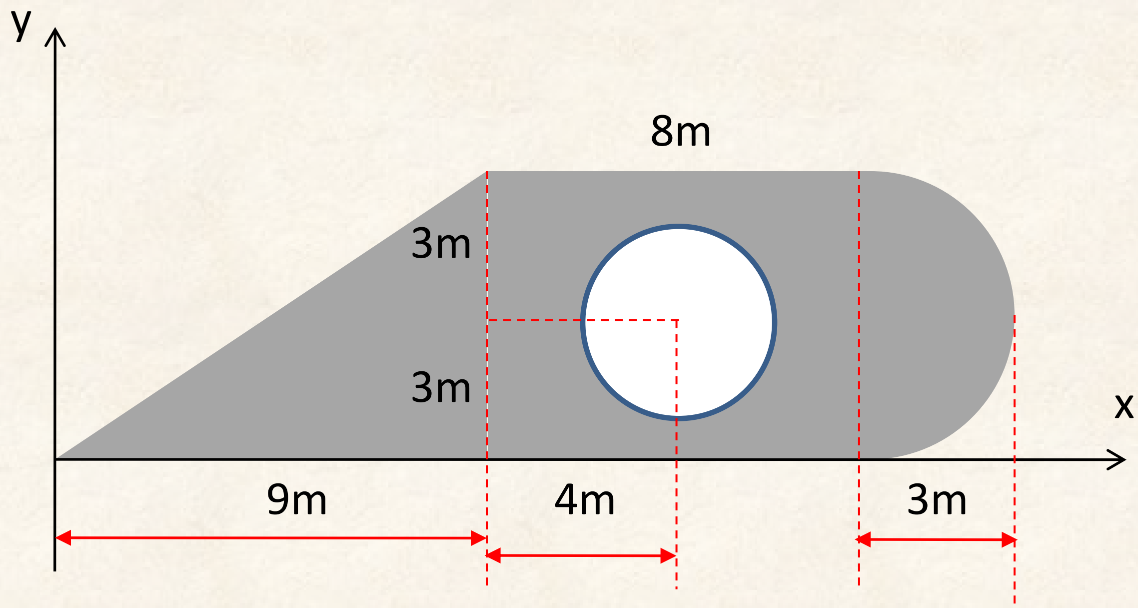
x_C	y_C	A
$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2}hb$
$\frac{a}{2}$	$\frac{b}{2}$	ab
r	r	πr^2



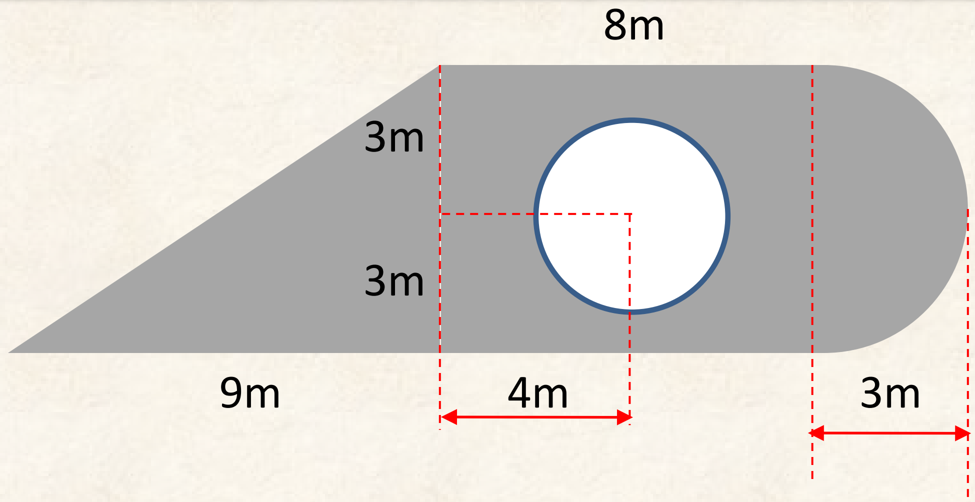
$$A = \frac{1}{4}\pi r^2$$

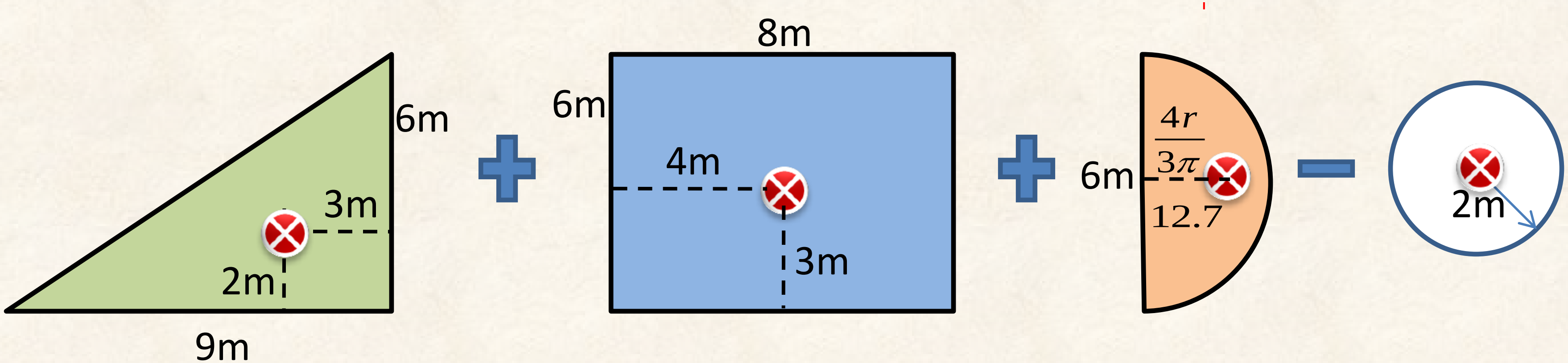
EXAMPLE-4

A circle of radius 2 m is removed from the uniform area shown. Determine the location of its centroid with respect to the x and y coordinates.

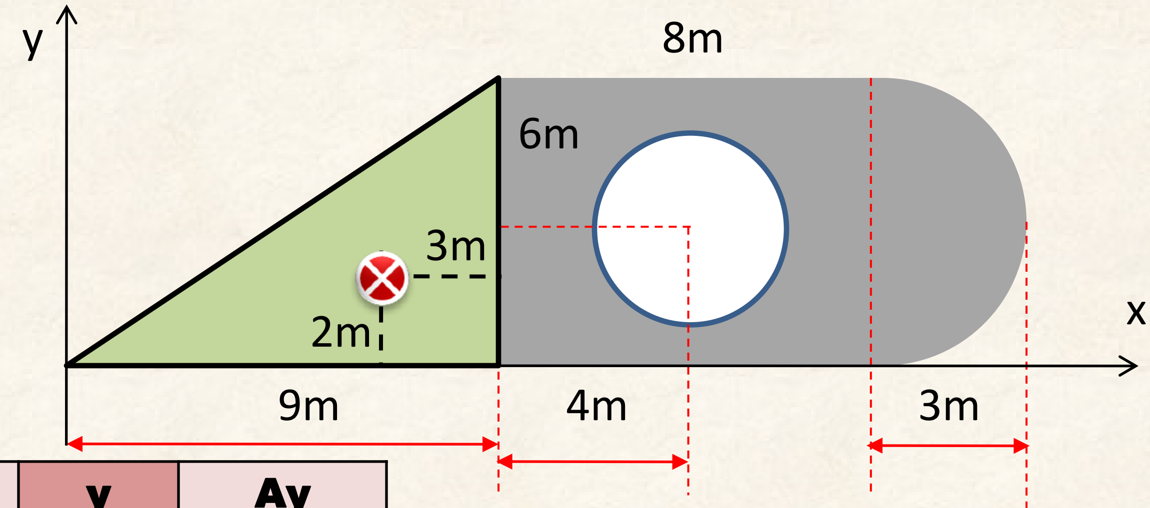


Solution



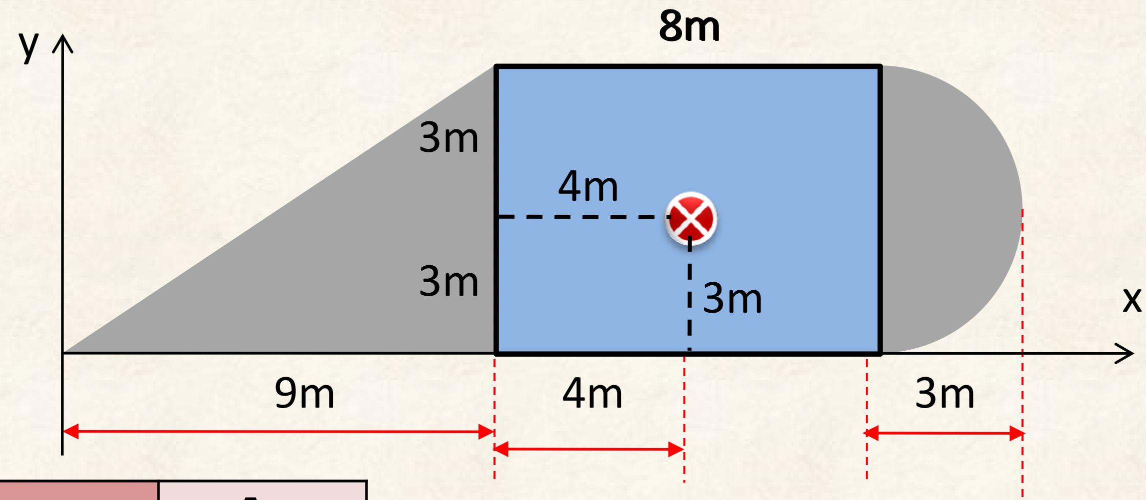


$A_1 = 27m^2$ $A_2 = 48m^2$ $A_3 = 14.14m^2$ $A_4 = -12.57m^2$



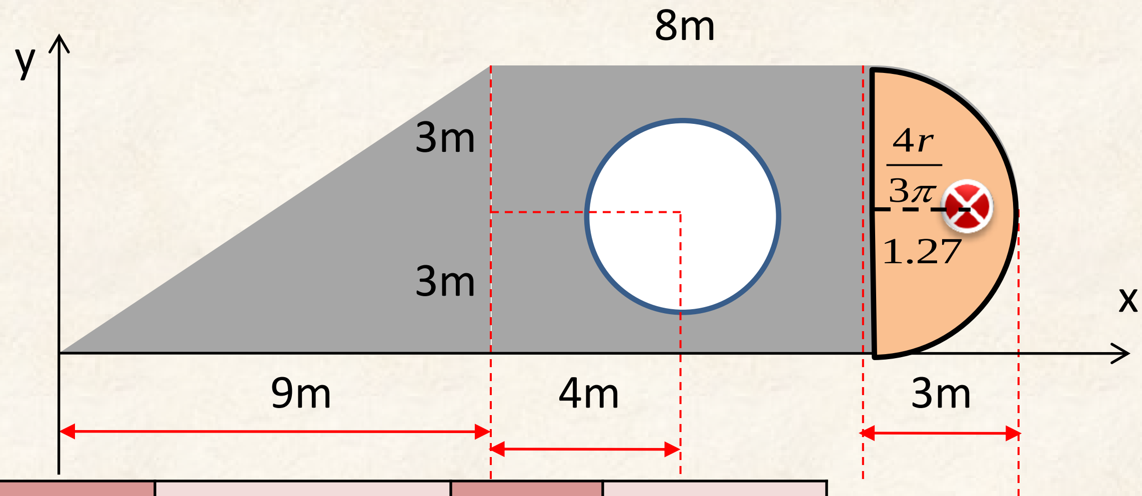
#	A	x	Ax	y	Ay
1	27	6		2	
2					
3					
4					
SUMS					

$$x_2 = 9 + 4 = 13\text{m}$$



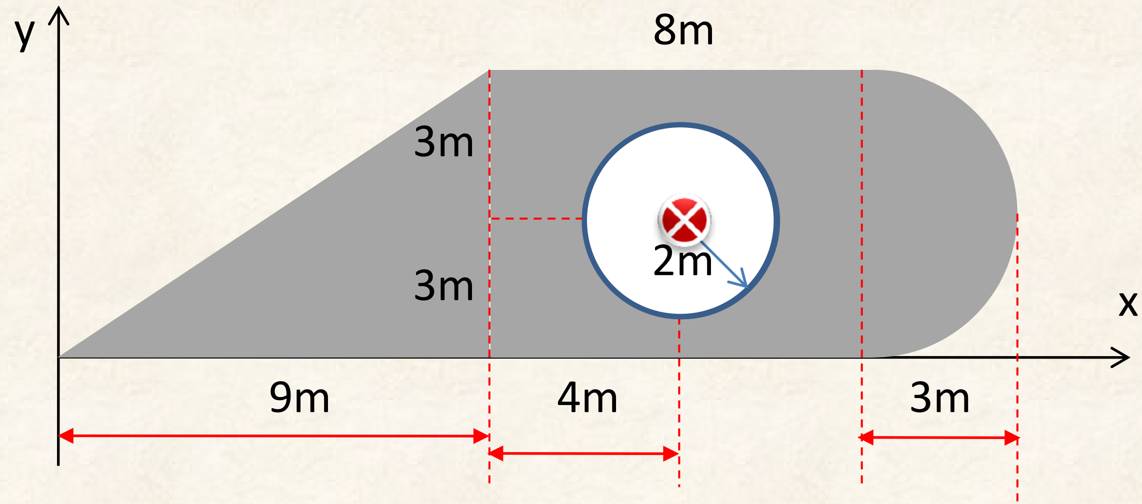
#	A	x	Ax	y	Ay
1	27	6		2	
2	48	13		3	
3					
4					
SUMS					

$$x_3 = 9 + 8 + 1.27 = 18.27\text{m}$$



#	A	x	Ax	y	Ay
1	27	6		2	
2	48	13		3	
3	14.14	18.27		3	
4					
SUMS					

$$x_4 = 9 + 4 = 13\text{m}$$



#	A	x	Ax	y	Ay
1	27	6		2	
2	48	13		3	
3	14.14	18.27		3	
4	-12.57	13		3	
SUMS					

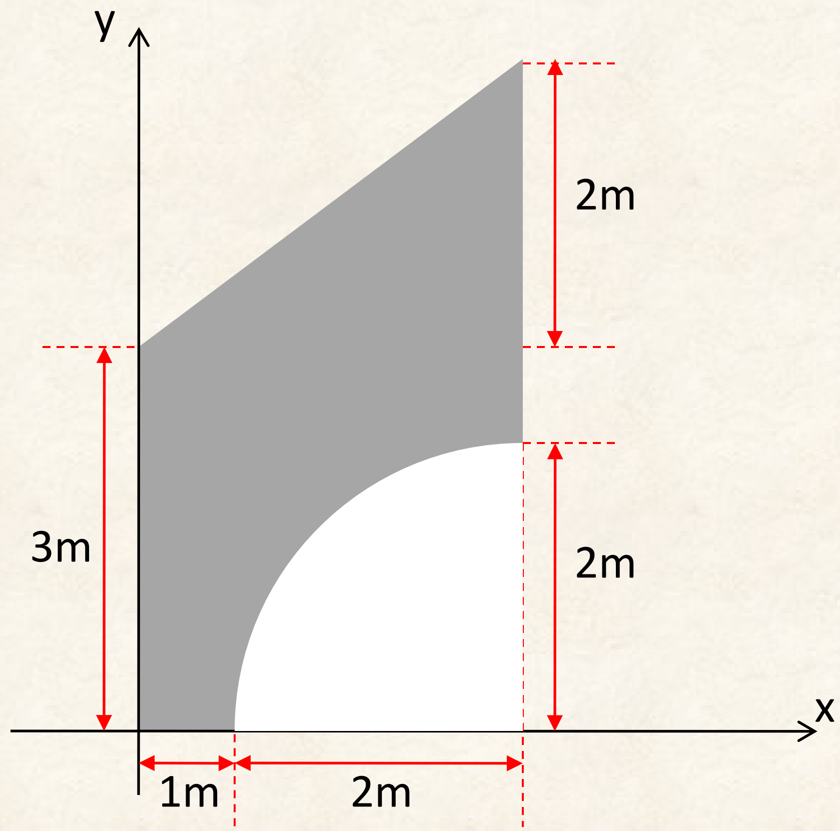
#	A	x	Ax	y	Ay
1	27	6	162	2	54
2	48	13	624	3	144
3	14.14	18.27	258.34	3	42.42
4	−12.57	13	−163.41	3	−37.71
SUMS	76.57		880.93		202.71

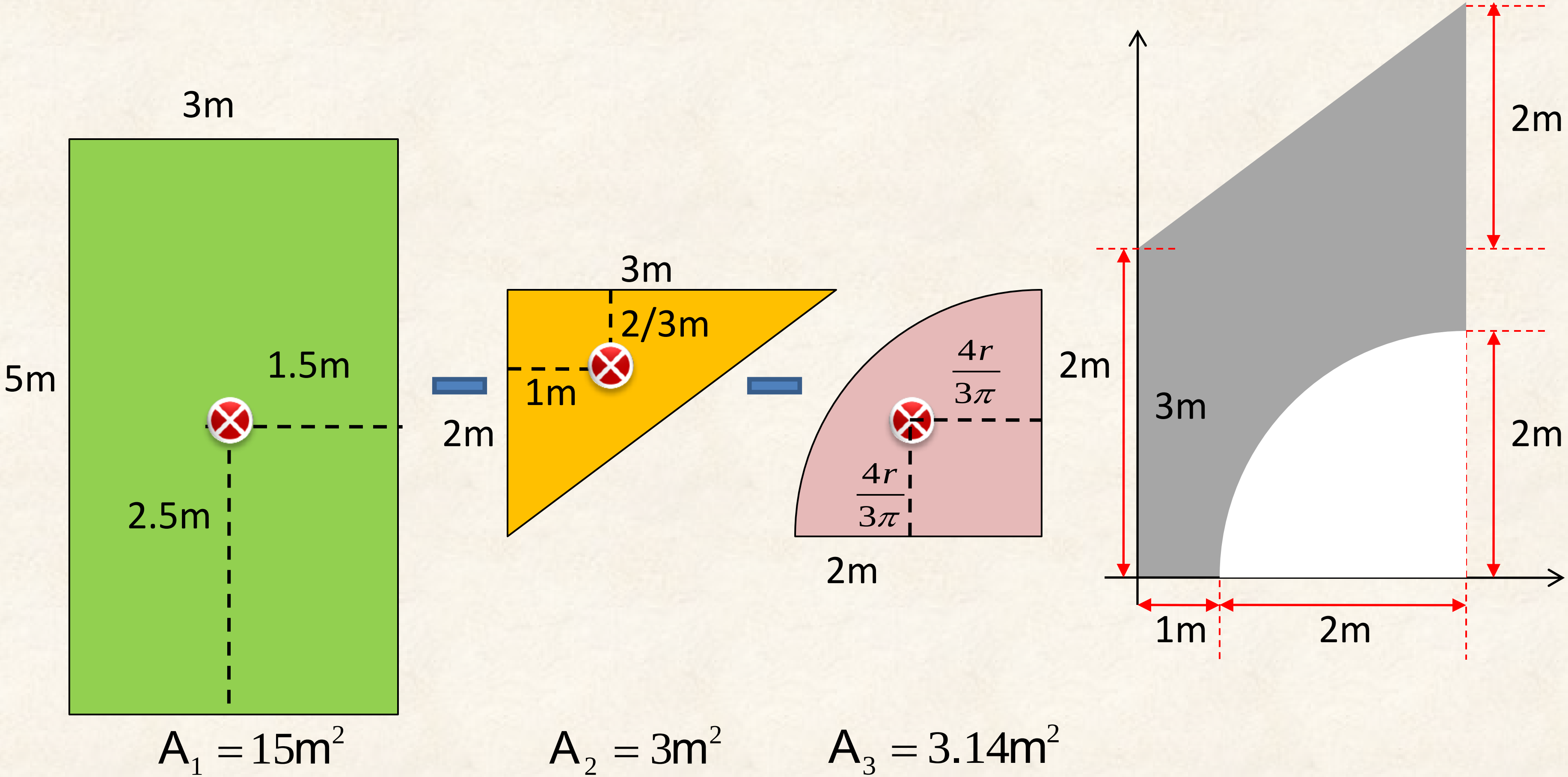
$$x_C = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} = \frac{880.93}{76.57} = 11.5\ m$$

$$y_C = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i} = \frac{202.71}{76.57} = 2.65\ m$$

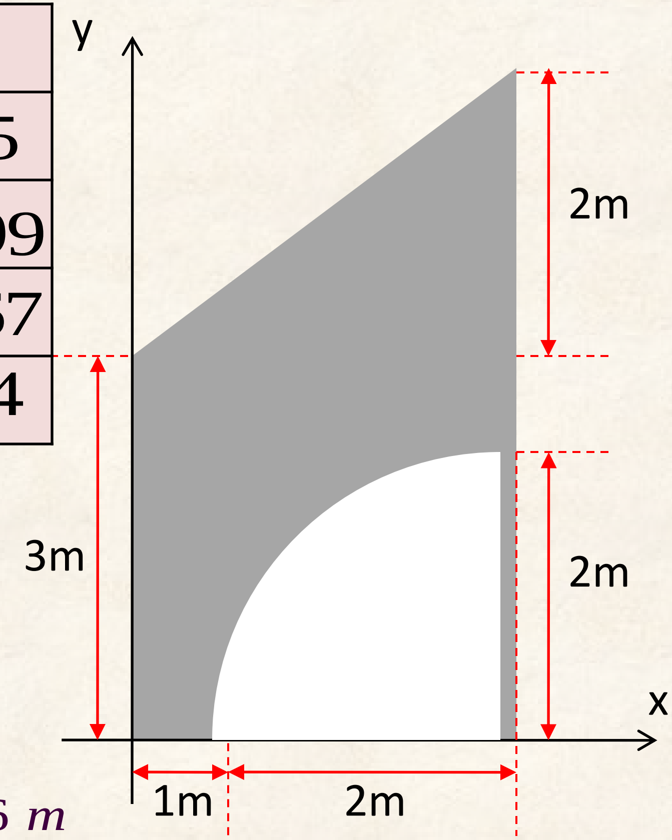
Example-5

Determine the location of the centroid with respect to the x and y coordinates of the uniform area shown.




3m
5m
1.5m
2.5m
$A_1 = 15m^2$
3m
2/3m
1m
2m
$A_2 = 3m^2$
2m
$\frac{4r}{3\pi}$
$\frac{4r}{3\pi}$
2m
$A_3 = 3.14m^2$
2m
3m
2m
1m
2m

#	A	x	Ax	y	Ay
1	15	1.5	22.5	2.5	37.5
2	–3	1	–3	4.33	–12.99
3	–3.14	2.15	–6.75	0.85	–2.67
SUMS	8.86		12.75		21.84

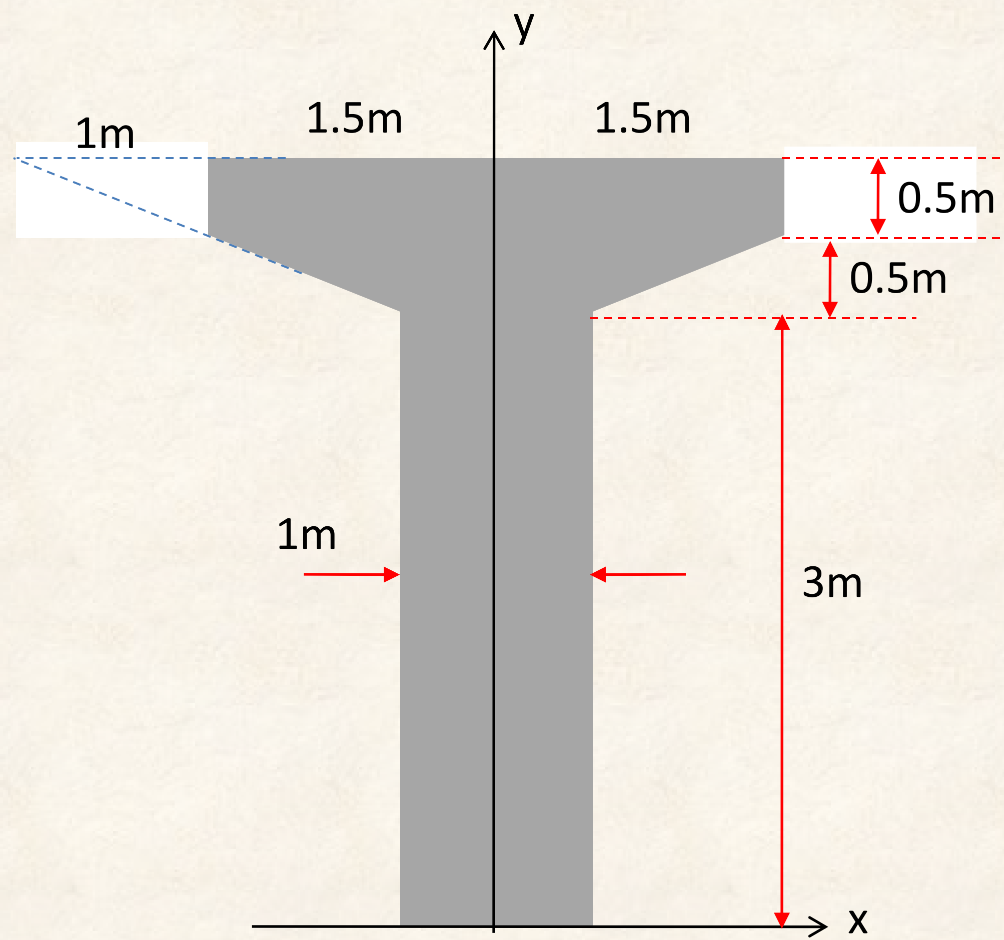


$$x_C = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} = \frac{12.75}{8.86} = 1.44\ m$$

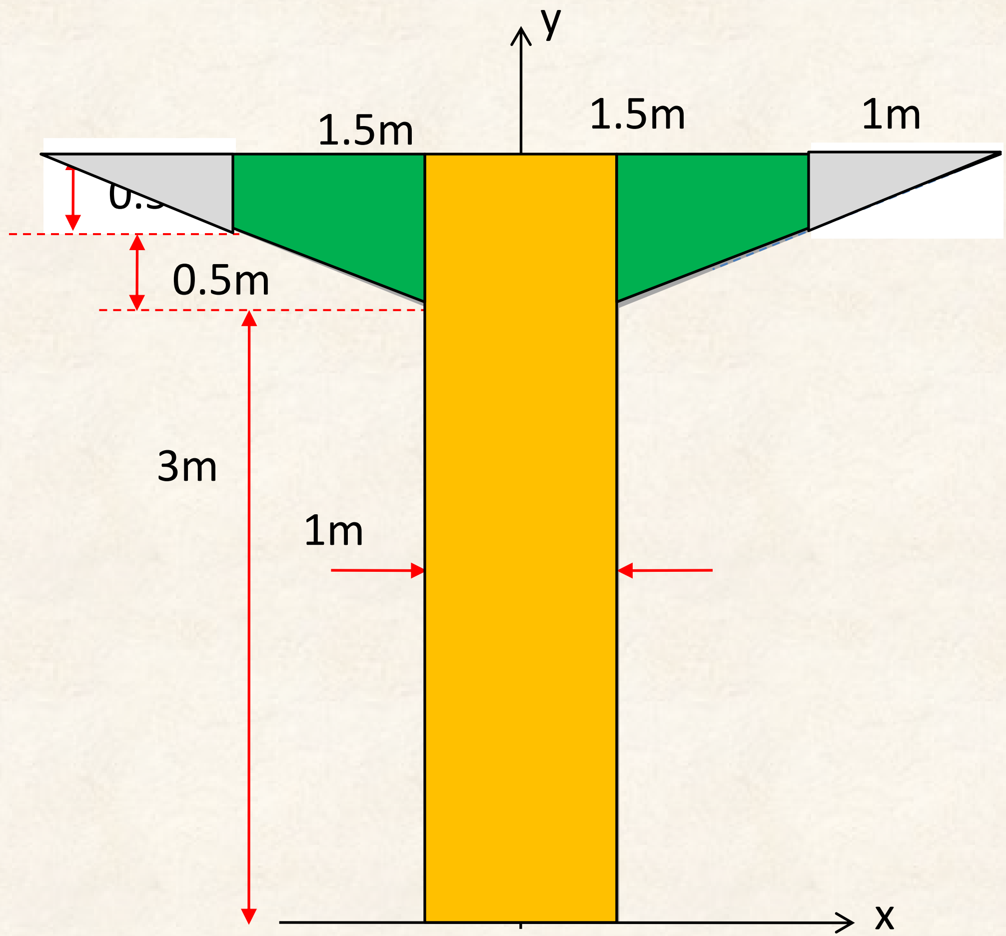
$$y_C = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i} = \frac{21.84}{8.86} = 2.46\ m$$

Home Work

Determine the location of the centroid with respect to the x and y coordinates of the uniform area shown.



Hint



MOMENTS OF INERTIA

Courage is the first of the human qualities because it is the quality which guarantees all the others.

Winston Churchill

CONTENTS

First moment of Area (Q)

Area Moments Of Inertia (I)

Polar Moments Of Inertia (J)

First moment of Area

The SI unit for **first moment of area** is a cubic metre (m^3). In the American Engineering and Gravitational systems the unit is a cubic foot (ft^3) or more commonly $inch^3$.

The **static** or **statical moment of area**, usually denoted by the symbol *Q*, is a property of a shape that is used to predict its resistance to shear stress. By definition:

$$Q_{j,x} = \int y_i \, dA,$$

where

$Q_{j,x}$ - the first moment of area "j" about the neutral *x* axis of the entire body (not the neutral axis of the area "j");

dA - an elemental area of area "j";

y - the perpendicular distance to the centroid of element *dA* from the neutral axis *x*.

Example

symm...

When the centroid C of an area can be located by symmetry, the first moment of that area with respect to any given axis can be readily obtained from Eqs. (A.4). For example, in the case of the rectangular area of Fig. A.6, we have

$$Q_x = A\bar{y} = (bh)(\tfrac{1}{2}h) = \tfrac{1}{2}bh^2$$

and

$$Q_y = A\bar{x} = (bh)(\tfrac{1}{2}b) = \tfrac{1}{2}b^2h$$

In most cases, however, it is necessary to perform the integrations indicated in Eqs. (A.1) through (A.3) to determine the first moments and the centroid of a given area. While each of the integrals involved is actually a double integral, it is possible in many applications to select elements of area dA in the shape of thin horizontal or vertical strips, and thus to reduce the computations to integrations in a single variable. This is illustrated in Example A.01. Centroids of common geometric shapes are indicated in a table inside the back cover of this book.

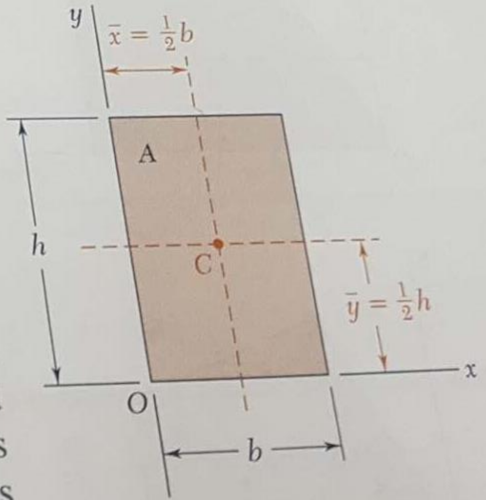


Fig. A.6

Example

EXAMPLE A.02

Locate the centroid C of the area A shown in Fig. A.10.

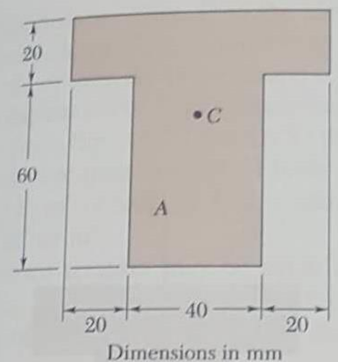


Fig. A.10

Selecting the coordinate axes shown in Fig. A.11, we note that the centroid C must be located on the y axis, since this axis is an axis of symmetry; thus, $\overline{X} = 0$.

Dividing A into its component parts A_1 and A_2, we use the second of Eqs. (A.6) to determine the ordinate $\overline{Y}$ of the centroid. The actual computation is best carried out in tabular form.

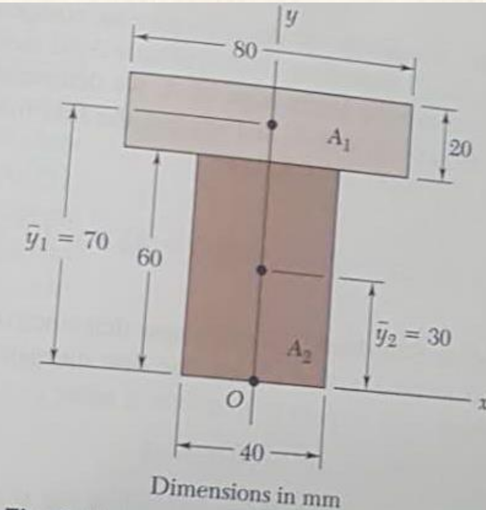


Fig. A.11

	Area, mm²	$\bar{y}_i$, mm	$A_i\bar{y}_i$, mm³
A_1	$(20)(80) = 1600$	70	112×10^3
A_2	$(40)(60) = 2400$	30	72×10^3
	$\sum_i A_i = 4000$		$\sum_i A_i\bar{y}_i = 184 \times 10^3$

$$\overline{Y} = \frac{\sum_i A_i\bar{y}_i}{\sum_i A_i} = \frac{184 \times 10^3 \text{ mm}^3}{4 \times 10^3 \text{ mm}^2} = 46 \text{ mm}$$

Example

EXAMPLE A.03

Referring to the area A of Example A.02, we consider the horizontal x' axis through its centroid C. (Such an axis is called a *centroidal axis*.) Denoting by A' the portion of A located above that axis (Fig. A.12), determine the first moment of A' with respect to the x' axis.

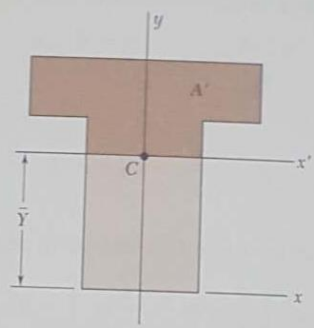


Fig. A.12

Solution. We divide the area A' into its components A_1 and A_3 (Fig. A.13). Recalling from Example A.02 that C is located 46 mm above the lower edge of A, we determine the ordinates $\bar{y}'_1$ and $\bar{y}'_3$ of A_1 and A_3 and express the first moment $Q'_{x'}$ of A' with respect to x' as follows:

$$Q'_{x'} = A_1\bar{y}'_1 + A_3\bar{y}'_3$$
$$= (20 \times 80)(24) + (14 \times 40)(7) = 42.3 \times 10^3 \text{ mm}^3$$

Area Moments of Inertia
(Second Moment of Area)

Area Moment of Inertia

The Four Area Moments of Inertia

Parallel Axes Theorem

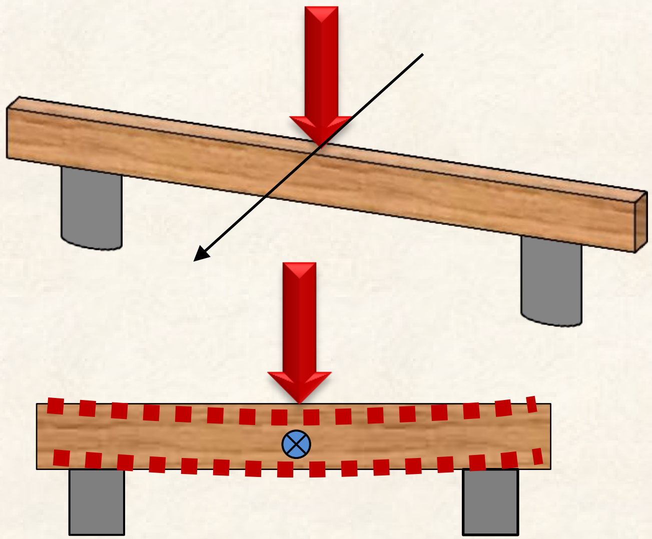
Evaluating Area Moment of Inertia for Composite Shapes

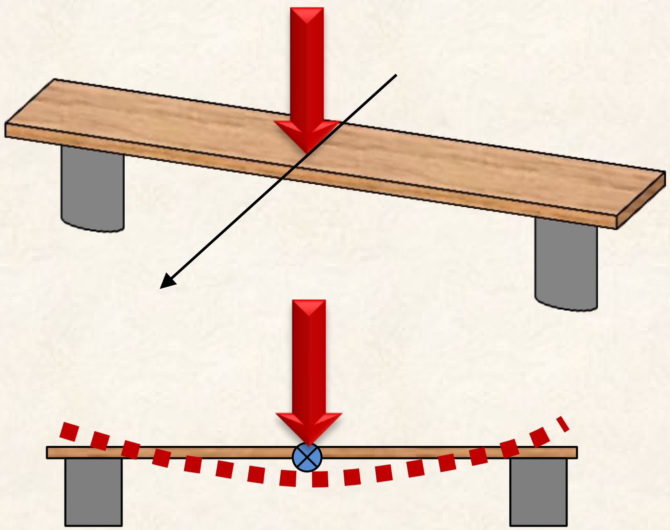
Area Moment of Inertia

Also known as the **second moment of area** or **second moment of inertia**

Measures the resistance of beams to bending and deflection.

The deflection of a beam under load depends not only on the load, but also on the geometry of the beam's cross-section





Four area moments of inertia defined for a beams cross section

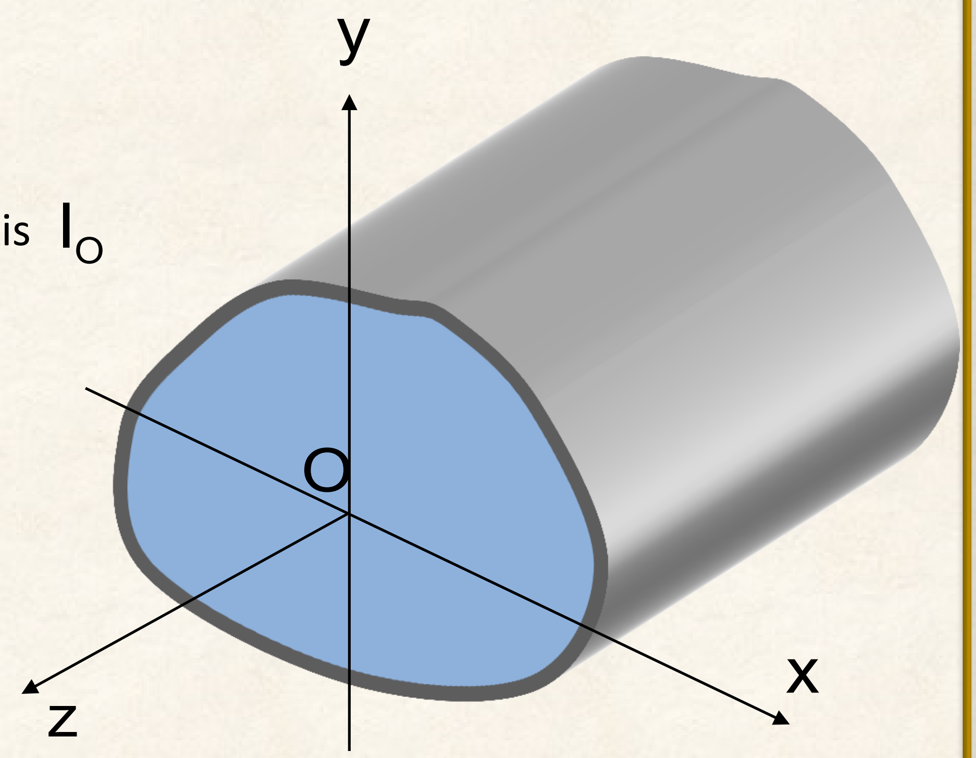
Area moment of inertia

about the x axis I_x

about the y axis I_y

Polar area moment of inertia about the z axis I_O

Area product moment of inertia I_{xy}

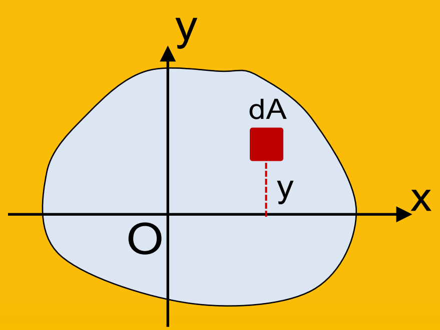


Area Moment of Inertia About the x-Axis

By definition

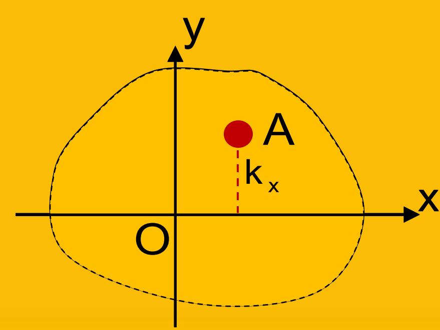
Using the integration method

$$I_x = \int y^2 dA$$



Using the radius of gyration

$$I_x = k_x^2 A$$



I_x measures the beams ability to resist bending about the x axis. The larger the Moment of Inertia the less the beam will bend.

The radius of gyration is the distance *k* away from the x axis that all the area can be concentrated to result in the same moment of inertia.

I_x is always positive

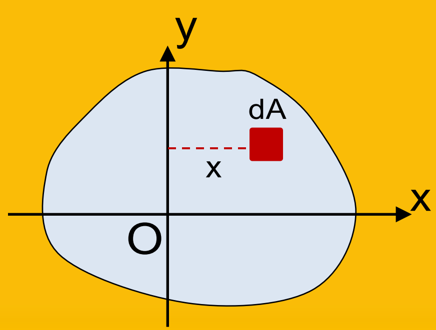
Units of I_x are m^4

Area Moment of Inertia About the y-Axis

By definition

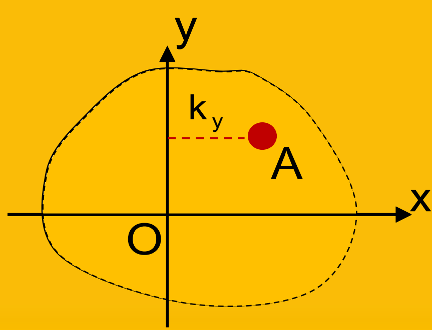
Using the integration method

$$I_y = \int x^2 dA$$



Using the radius of gyration

$$I_y = k_y^2 A$$

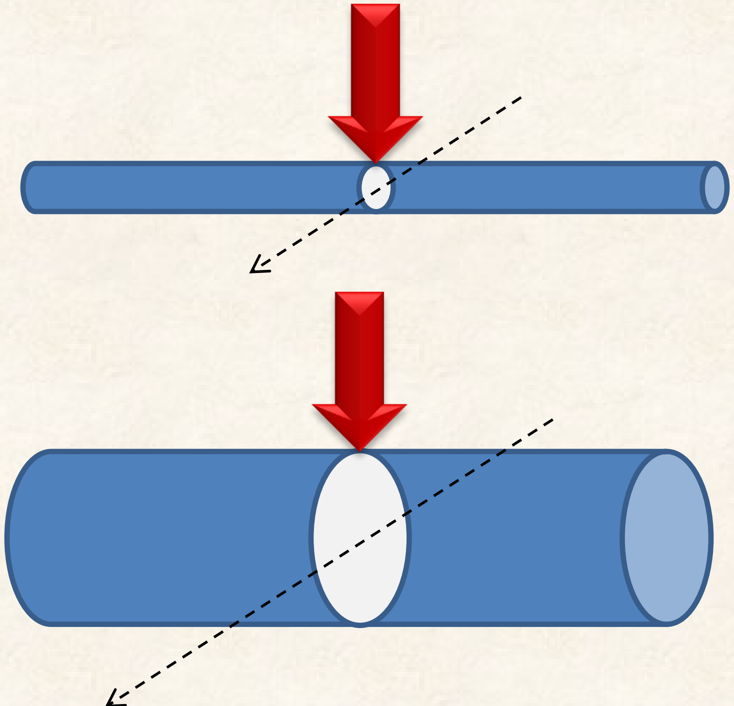


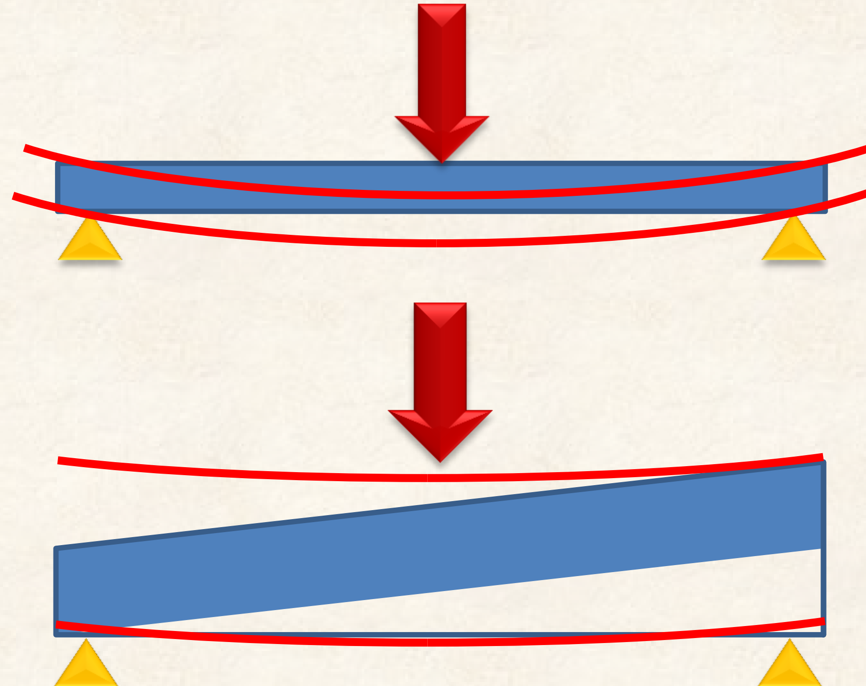
I_y measures the beams ability to resist bending about the y axis. The larger the Moment of Inertia the less the beam will bend.

The radius of gyration is the distance *k* away from the y axis that all the area can be concentrated to result in the same moment of inertia.

I_y is always positive

Units of I_y are m^4

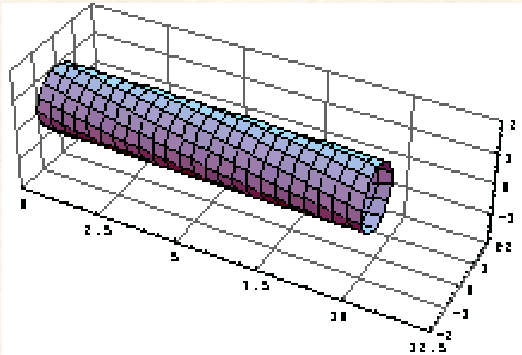


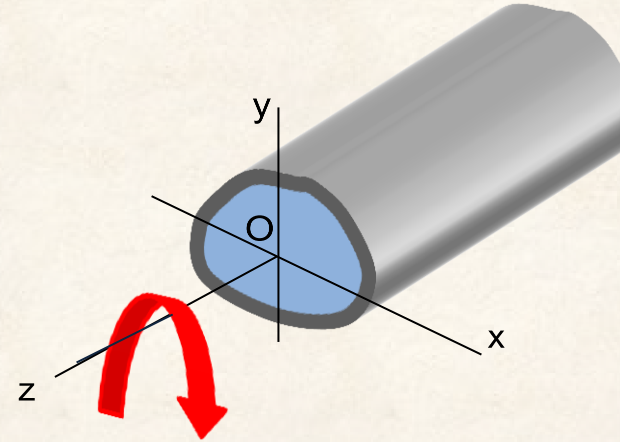



builderbill

Polar Area Moment of Inertia About the z Axes

The Polar Area Moment of Inertia of a beams cross-sectional area measures the beams ability to resist torsion. The larger the Polar Moment of Inertia the less the beam will twist.



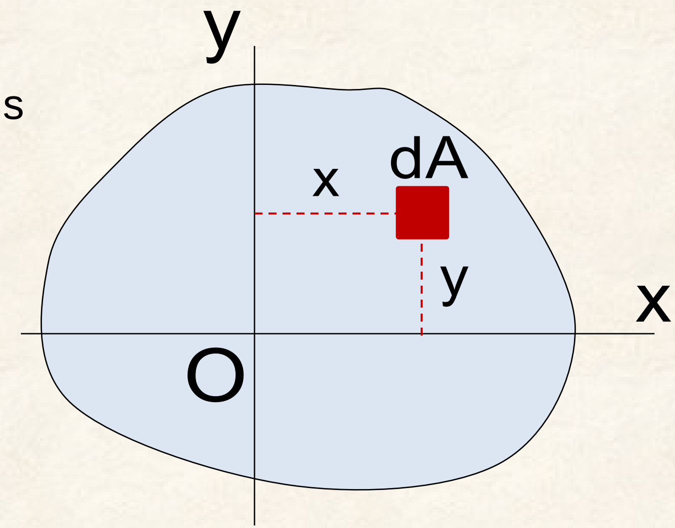


$$I_o = I_x + I_y = k_x^2 A + k_y^2 A = \left(k_x^2 + k_y^2\right)A = k_o^2 A$$

Product Area Moment of Inertia

The product of inertia for an area A about the x and y axes is defined as

$$I_{xy} = \int xy dA$$



Clearly from the definition of the product of inertia, we could reverse indices, but still both quantities are equal, i.e.

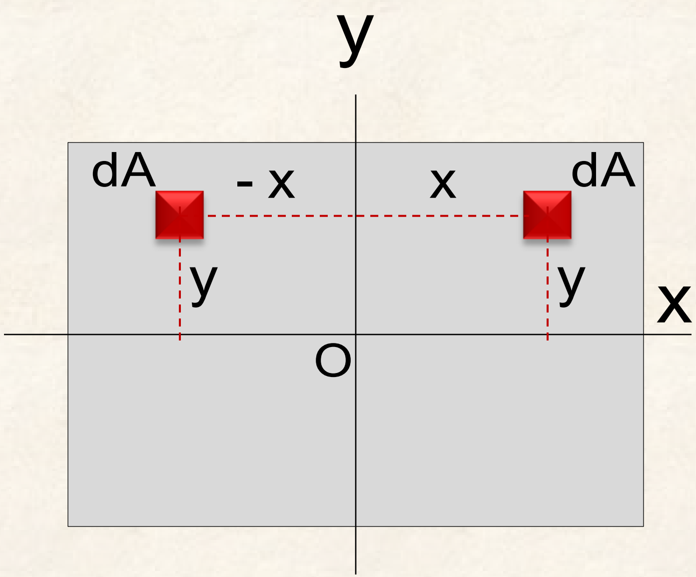
$$I_{xy} = I_{yx}$$

It is seen that I_{xy} can either be positive or negative depending on the signs of the x and y locations with respect to the element area dA.

The manner in which the area of a planar figure is situated in the coordinate quadrants is described by the area product of inertia. Because I_{xy} is positive if an element of area is located in the first or third quadrant, and negative in the second or fourth quadrant, we conclude that positive I_{xy} means that area predominates in the first and/or third quadrants. Obviously, when I_{xy} is negative, the area predominates in the second and/or fourth quadrants.

Principle Axes

In the important case where a shape is symmetric about one of the coordinate axes, the value of xy for an element to one side of the axis of symmetry is canceled by the value xy for the mirror-image element to the other side.. Thus whenever a planar shape has an axis of symmetry that is either the x or y axis, then



$$I_{xy} = 0$$

In general, when the x and y coordinates axes give a zero product of inertia we say that they are principal axes.

Summary

Name	Symbol	Formula Integration	Formula Radius of gyration	sign	Measures
Area Moment of inertia about the x axis	$I_x = I_{xx}$	$I_x = \int y^2 dA$	$I_x = k_x^2 A$	Always positive	Bending about the x axis. The larger the moment of inertia, the less the beam will bend.
Area Moment of inertia about the y axis	$I_y = I_{yy}$	$I_y = \int x^2 dA$	$I_y = k_y^2 A$	Always positive	Bending about the y axis. The larger the moment of inertia, the less the beam will bend.
Polar area moment of inertia about the z axis	$I_O = I_{zz}$		$I_O = k_O^2 A = I_x + I_y$	Always positive	Twisting (torsion) in beams. The larger the polar moment of inertia the less the beam will twist
Product area of inertia	I_{xy}	$I_{xy} = \int xy dA$		May be positive or negative	When the product of inertia is zero, at least one the x or y axes becomes a principle axis. Representing maximum or minimum moment of inertia

Parallel Axes Theorem

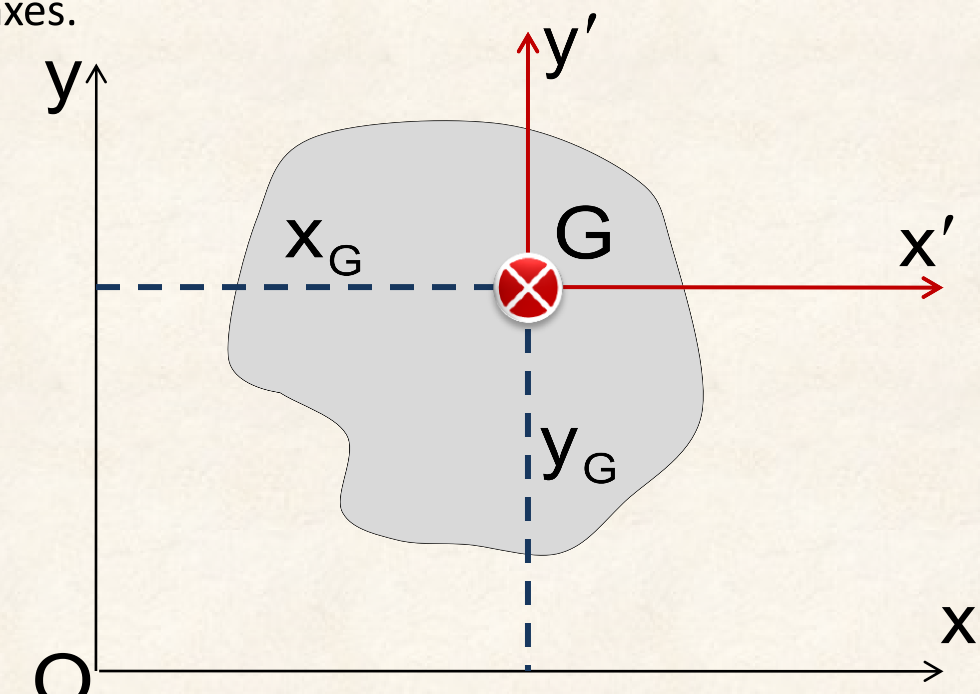
The **parallel axis theorem** can be used to determine the area moment of inertia about any axis, given the area moment of inertia about the parallel axis through the area's centroid and the perpendicular distance between the axes.

$$I_x = I_{x'} + A y_G^2$$

$$I_y = I_{y'} + A x_G^2$$

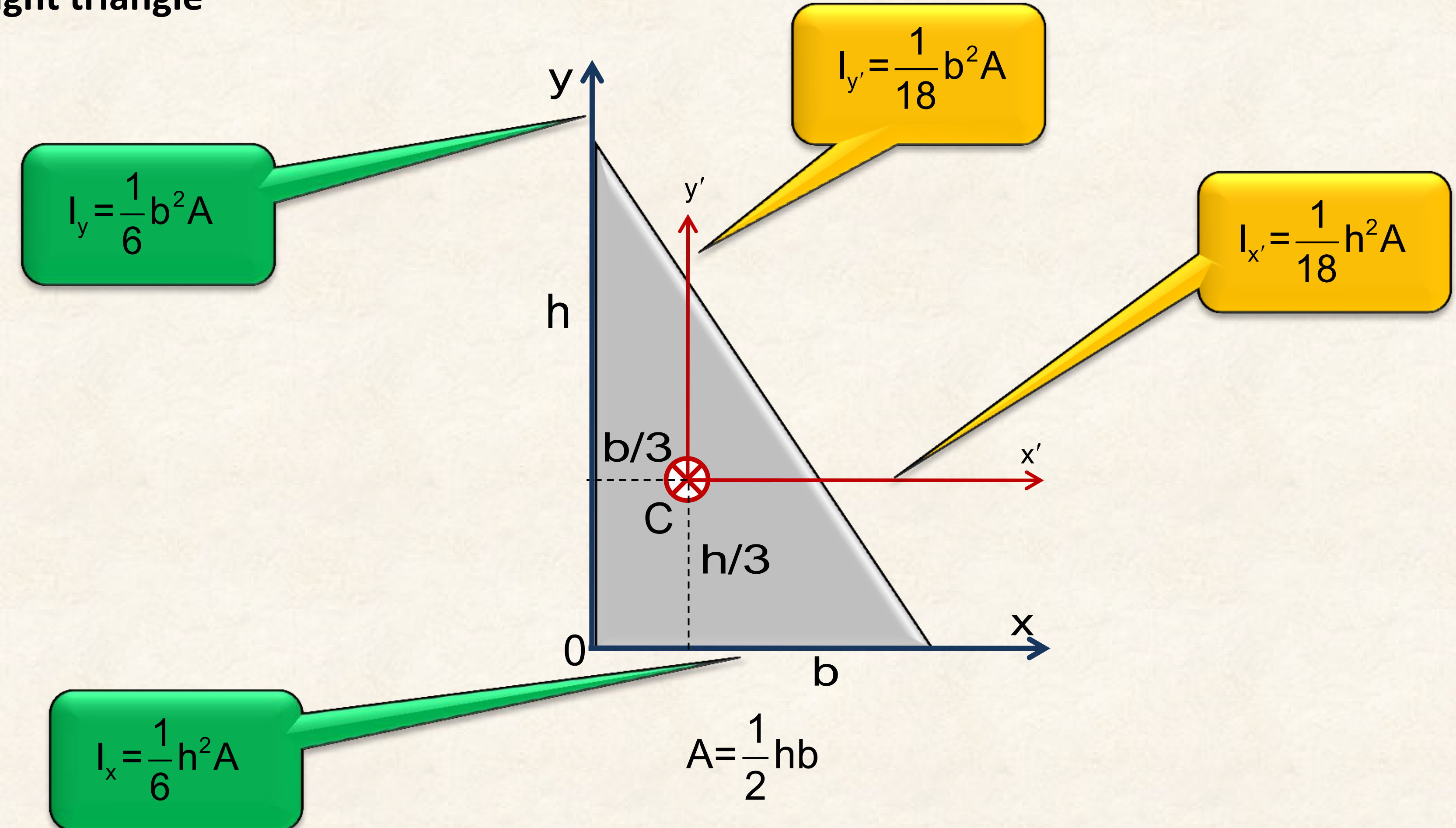
$$I_{xy} = I_{x'y'} + A x_G y_G$$

$$I_O = I_G + A\left(x_G^2 + y_G^2\right) = I_x + I_y$$



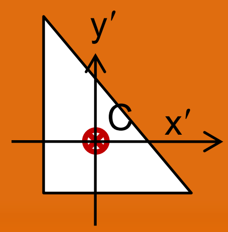
Area Moment of Inertia For Some Common Cross Sections

Right triangle

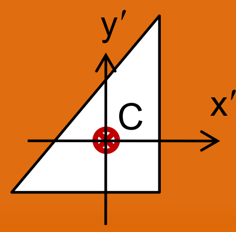


Right triangle

Product of inertia w.r.t. centroidal axes

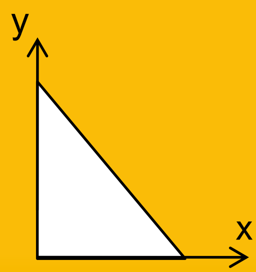


$$I_{x'y'} = \frac{-1}{36} hbA$$

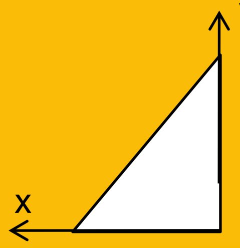


$$I_{x'y'} = \frac{1}{36} hbA$$

Product of inertia w.r.t. xy axes as sides

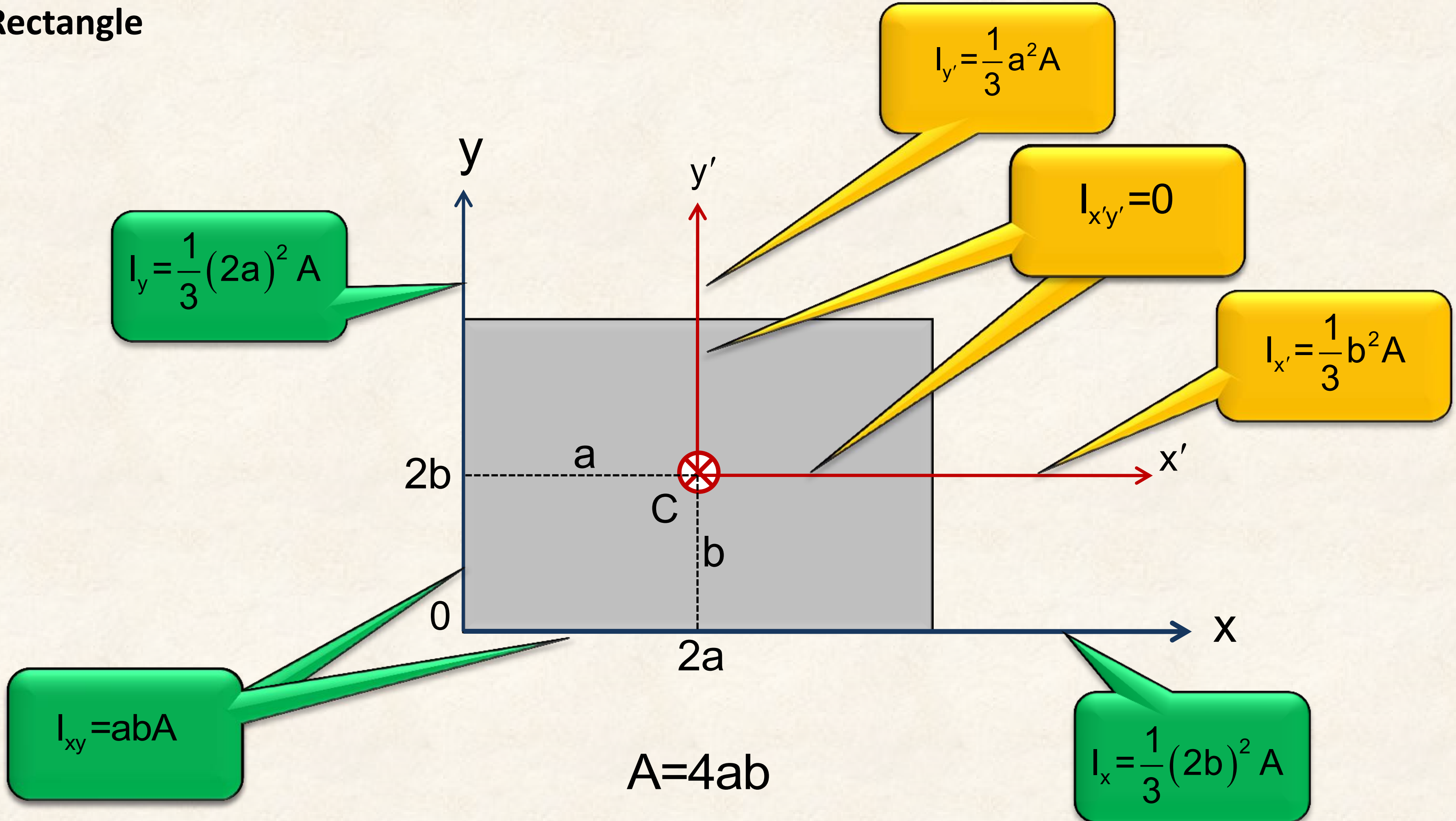


$$I_{xy} = \frac{1}{18} hbA$$



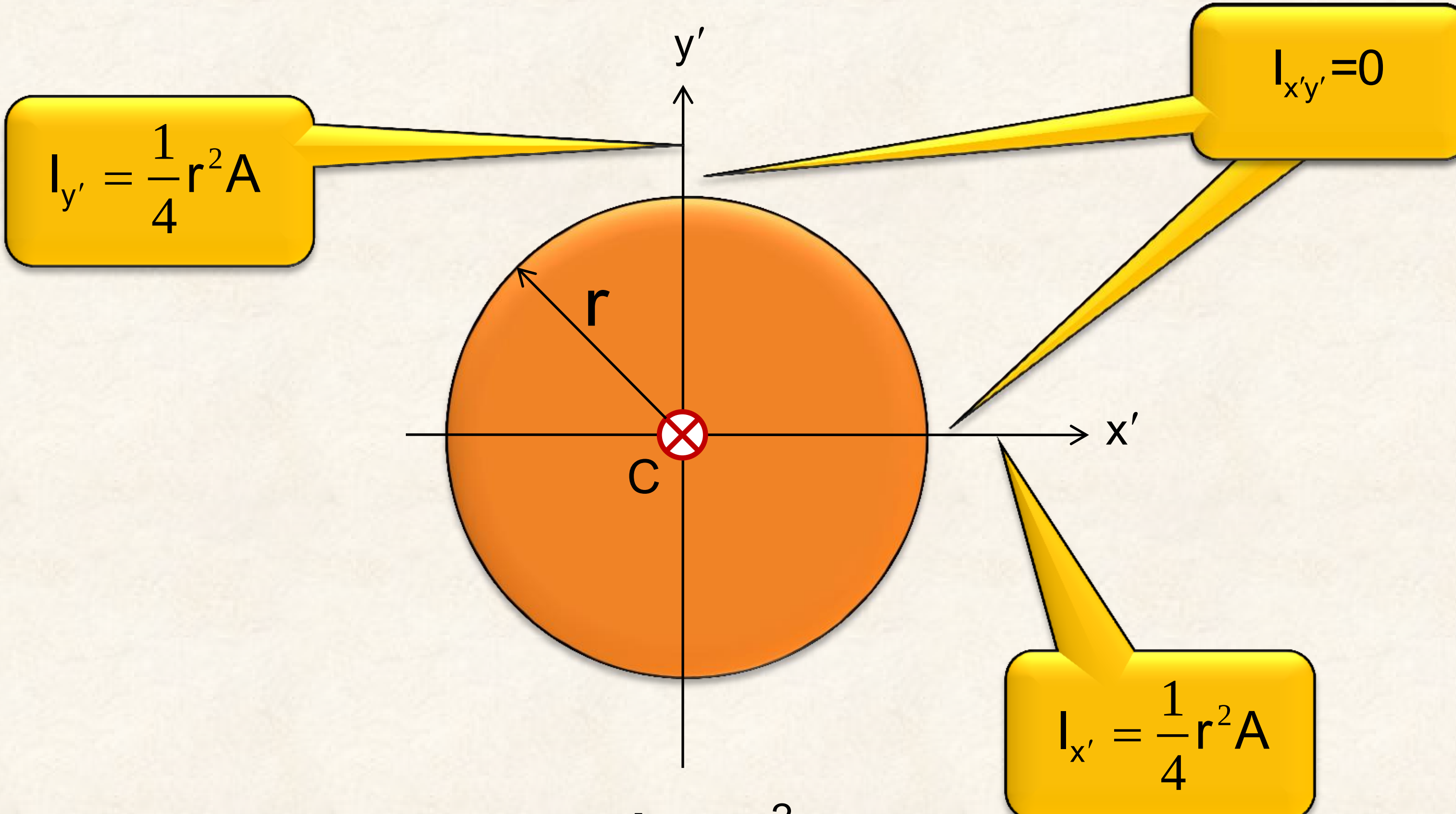
$$I_{xy} = \frac{-1}{18} hbA$$

Rectangle



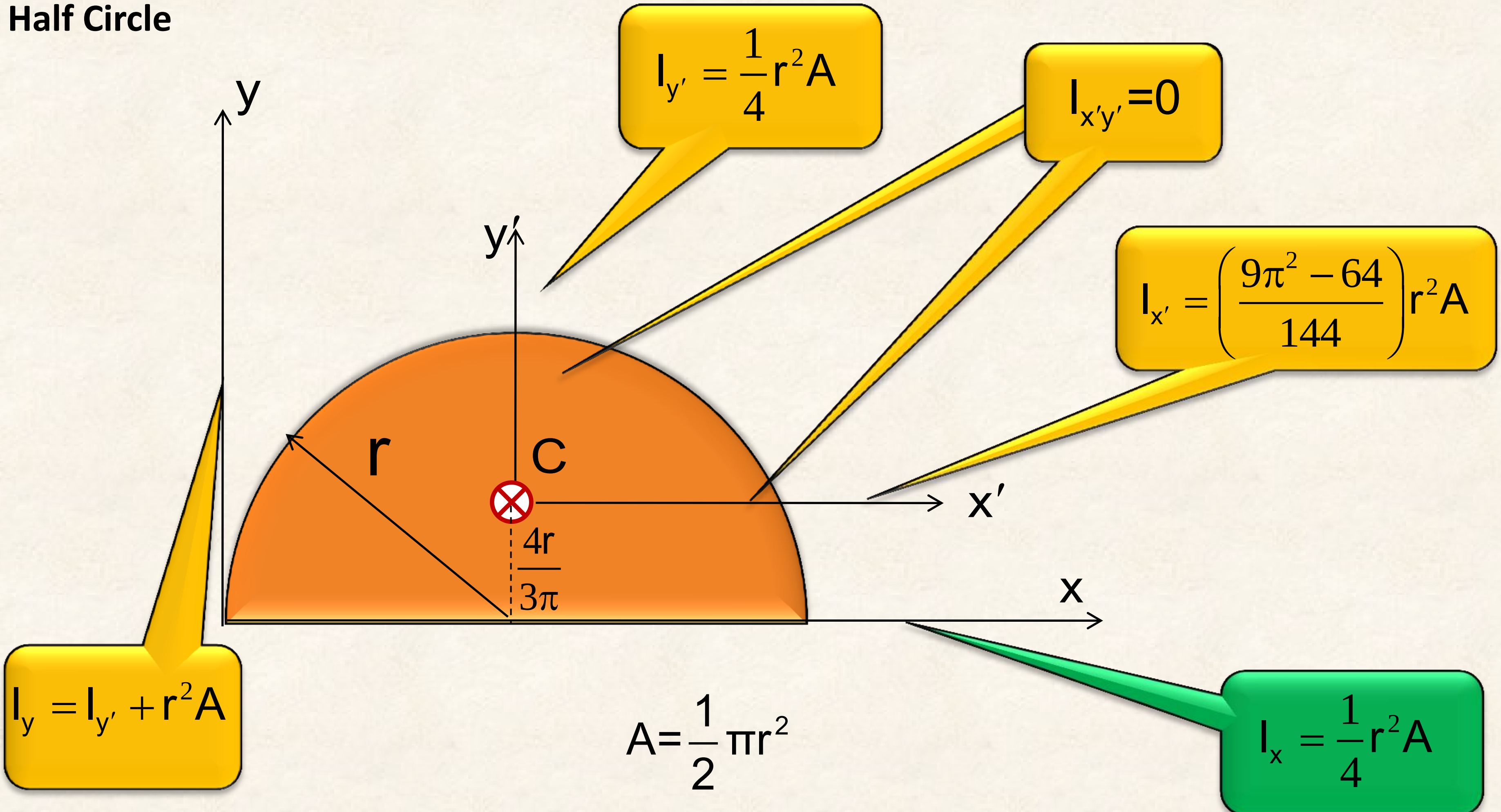
$I_{y'}=\frac{1}{3}a^2A$
$I_{x'y'}=0$
$I_{x'}=\frac{1}{3}b^2A$
$I_y=\frac{1}{3}(2a)^2A$
$I_{xy}=abA$
$I_x=\frac{1}{3}(2b)^2A$
y
y′
x
x′
2b
a
C
b
0
2a
A=4ab

Complete Circle



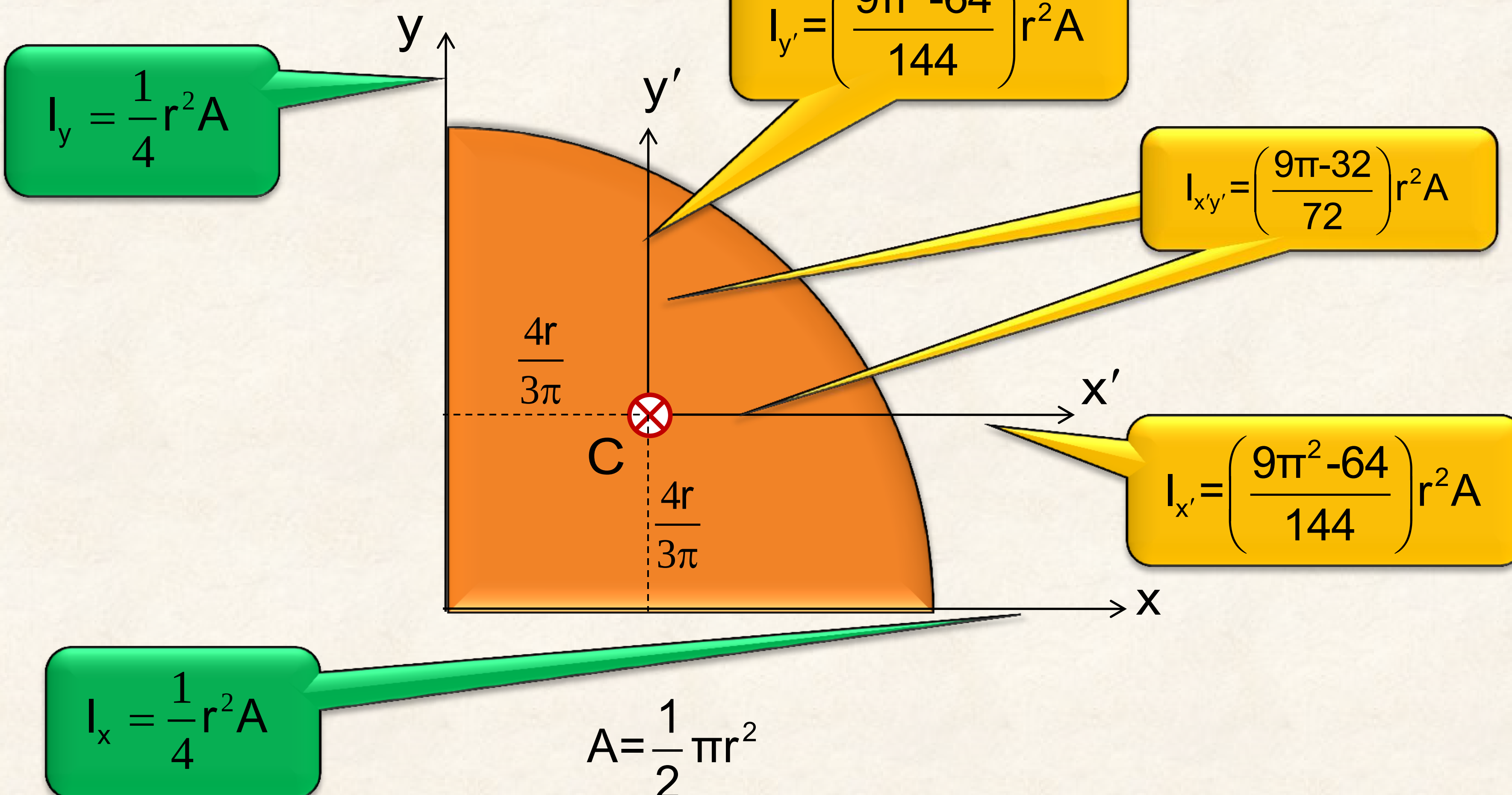
$A=\pi r^2$

Half Circle



$I_{y'} = \frac{1}{4} r^2 A$
$I_{x'y'} = 0$
$I_{x'} = \left(\frac{9\pi^2 - 64}{144} \right) r^2 A$
$I_y = I_{y'} + r^2 A$
$I_x = \frac{1}{4} r^2 A$
$A = \frac{1}{2} \pi r^2$
y
y′
x
x′
r
C
$\frac{4r}{3\pi}$

Quarter Circle

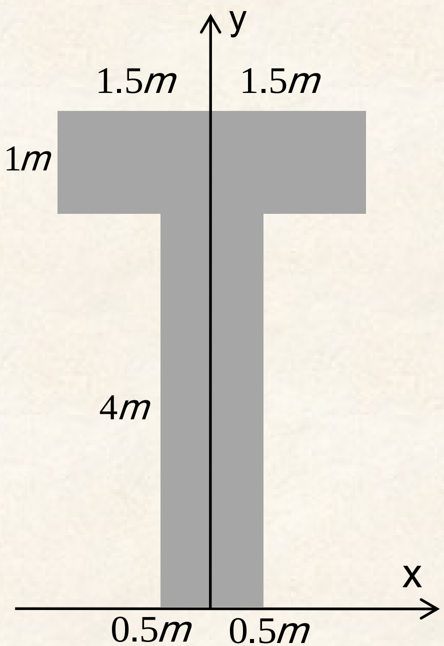


$$A = \frac{1}{2} \pi r^2$$

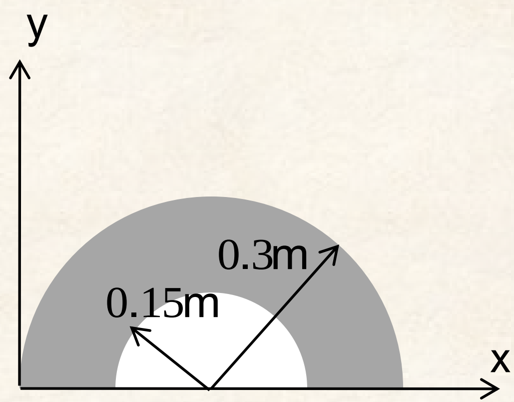
Example-1

i- Determine the moments and products of inertia for the cross sections shown about the x and y axes

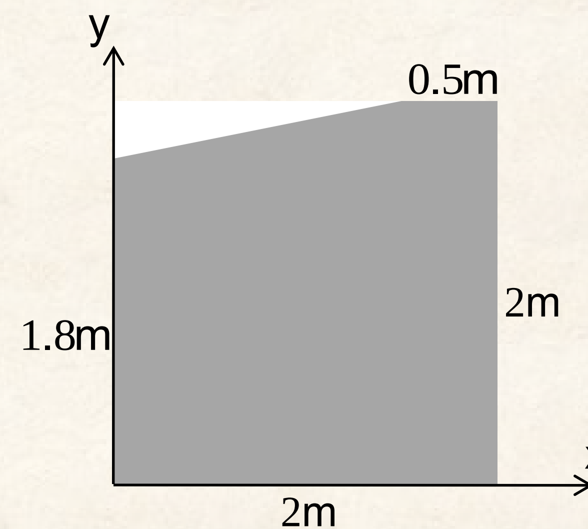
ii- Determine the moments and products of inertia for the cross sections (b) and (c) about their centroidal axes



(a)

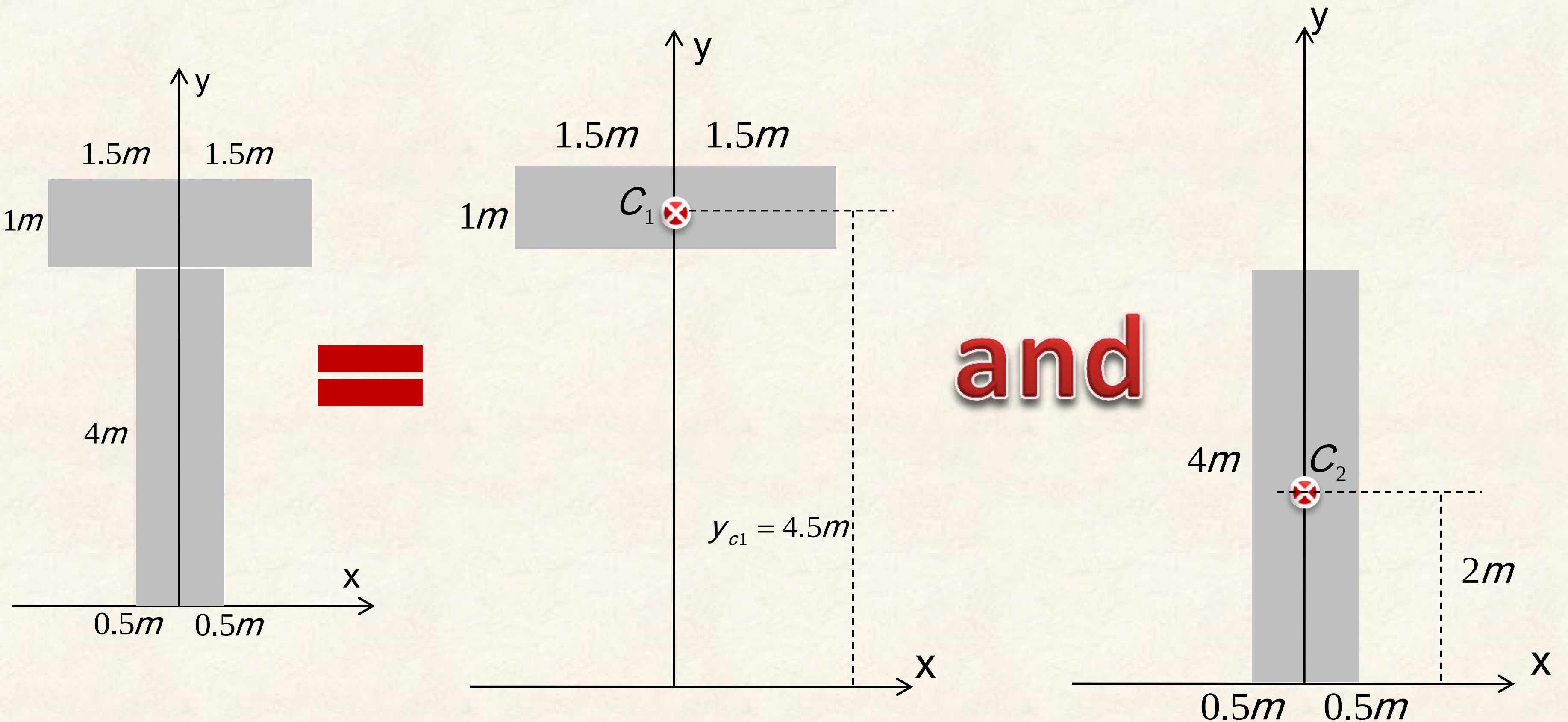


(b)

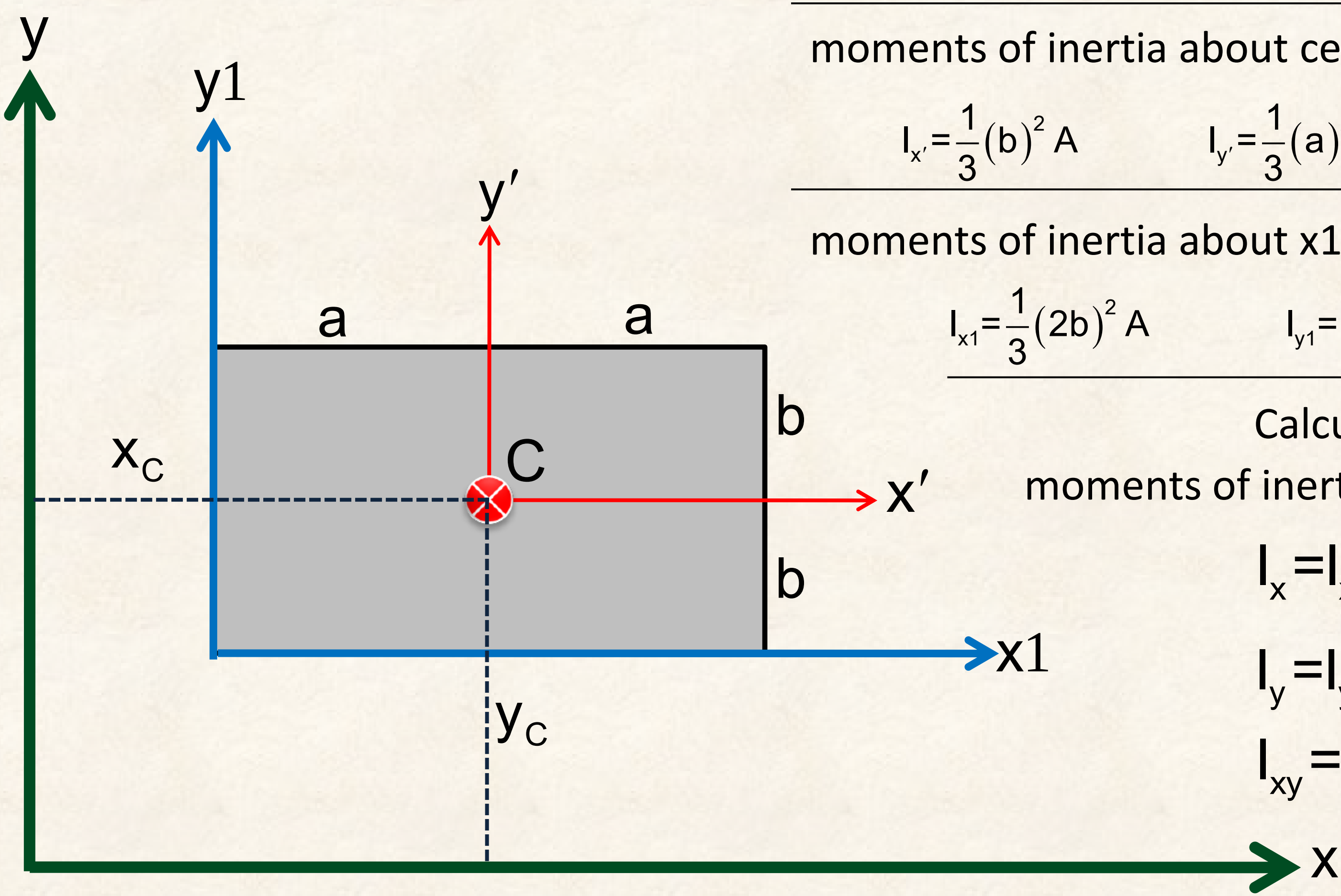


(c)

Solution : (cross section a)



Remember



Given data

area $A=4ab$

moments of inertia about centroidal axes

$$I_{x'}=\frac{1}{3}(b)^2 A \qquad I_{y'}=\frac{1}{3}(a)^2 A \qquad I_{x'y'}=0$$

moments of inertia about x1 and y1 axes

$$I_{x1}=\frac{1}{3}(2b)^2 A \qquad I_{y1}=\frac{1}{3}(2a)^2 A \qquad I_{x1y1}=abA$$

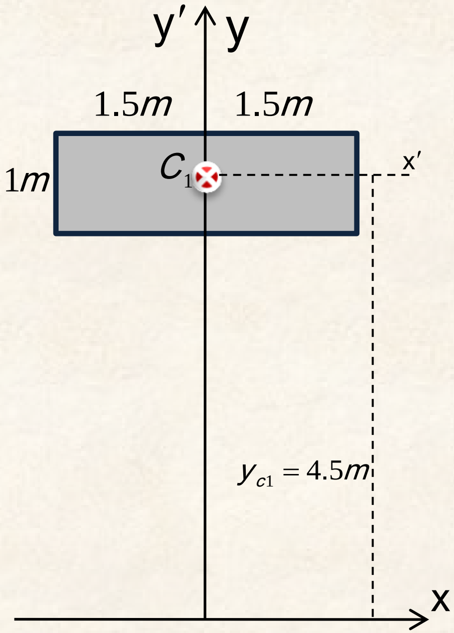
Calculated data

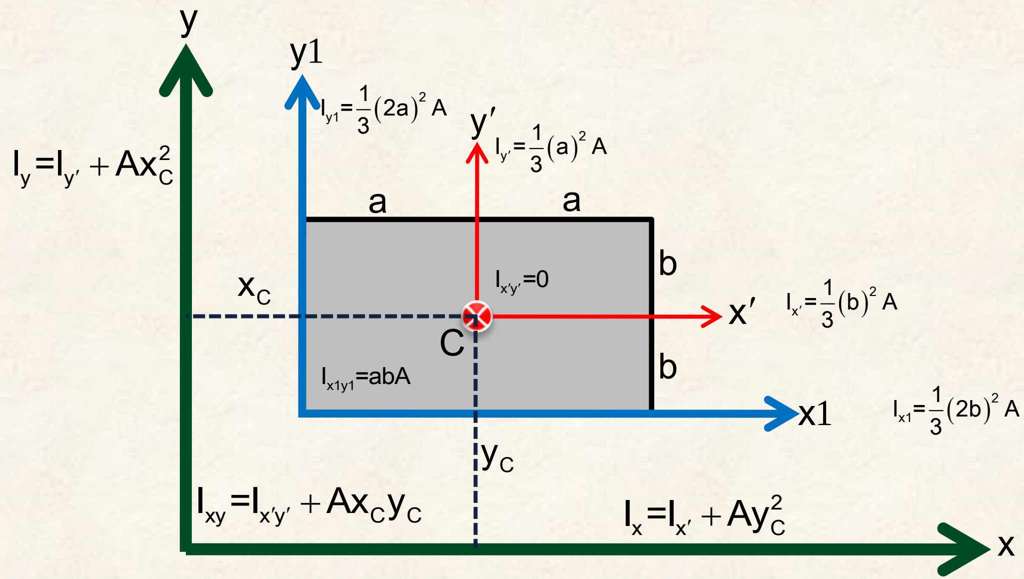
moments of inertia about x and y axes

$$I_x=I_{x'}+Ay_C^2$$

$$I_y=I_{y'}+Ax_C^2$$

$$I_{xy}=I_{x'y'}+Ax_Cy_C$$





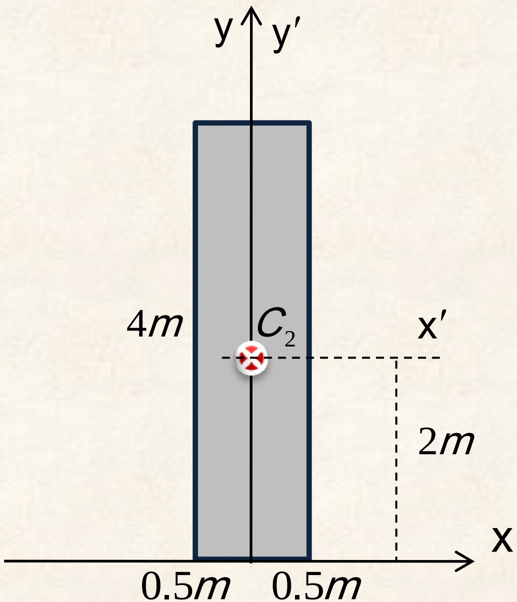
$$A_1 = 1x3 = 3m^2$$

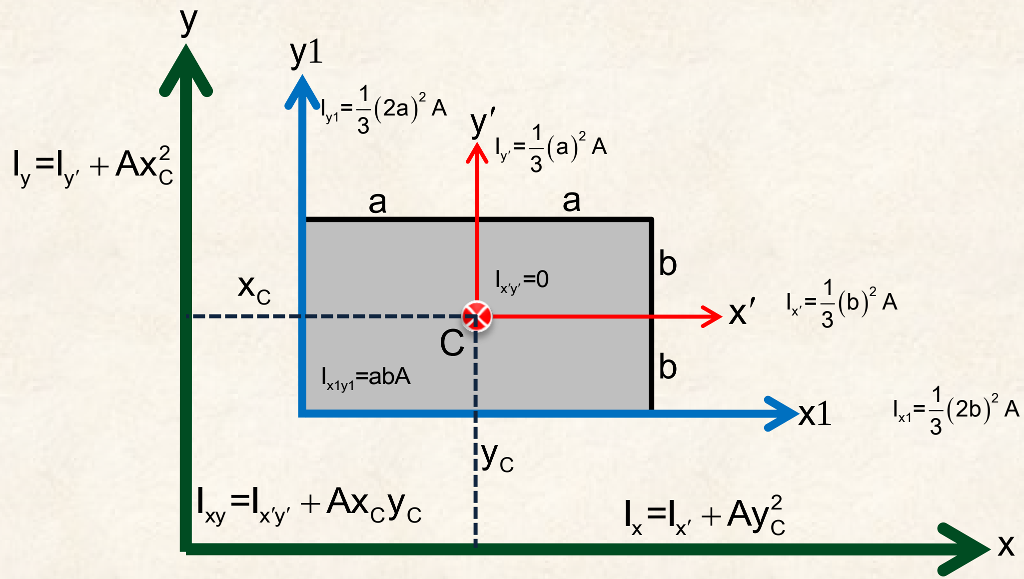
$$I_{x1} = I_{x'1} + A_1(y_{c1})^2$$

$$I_{x1} = \frac{1}{3}(0.5)^2(3) + 3(4.5)^2 = 61m^4$$

$$I_{y1} = \frac{1}{3}(1.5)^2(3) = 2.25m^4$$

$$I_{xy1} = 0$$





$$A_2=1x4=4m^2$$

$$I_{x2}=\frac{1}{3}(2b)^2A=\frac{1}{3}(4)^2 4=21.33m^4$$

$$I_{xy2}=0$$

$$I_{y2}=\frac{1}{3}(a)^2A=\frac{1}{3}(0.5)^2(4)=0.33m^4$$

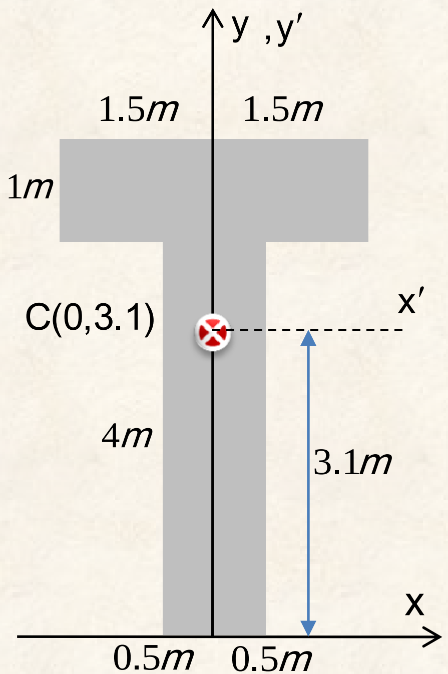
Total area moments of inertia

$$I_x = I_{x1} + I_{x2} = 61 + 21.33 = 82.33\,m^4$$

$$I_y = I_{y1} + I_{y2} = 2.25 + 0.33 = 2.58\,m^4$$

$$I_{xy} = I_{xy1} + I_{xy2} = 0$$

Moments of Inertia about the centroidal axes.



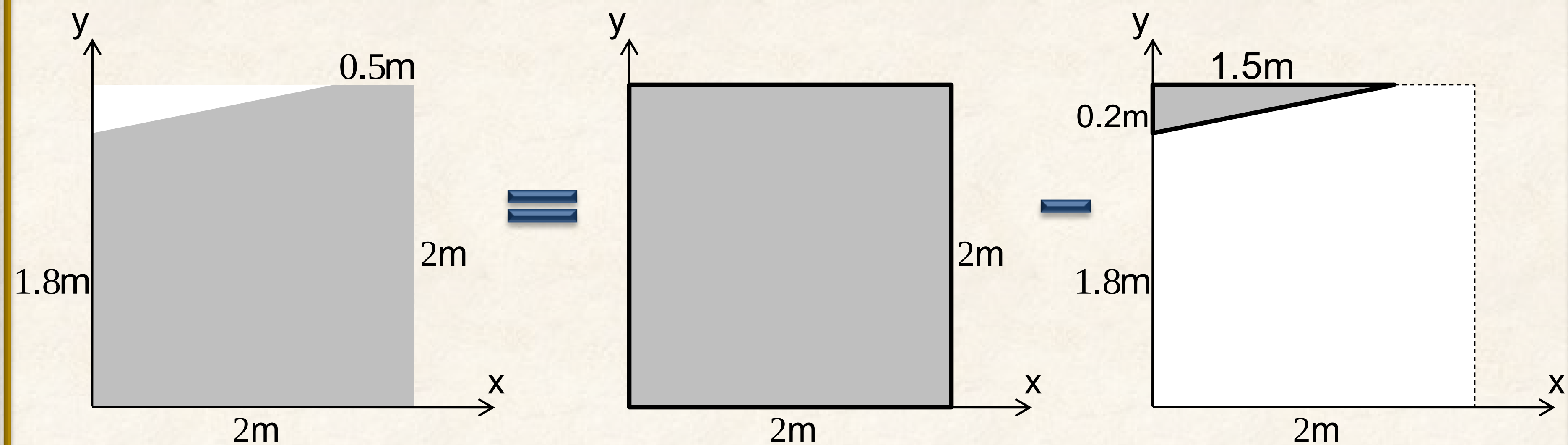
$$I_x = I_{x'} + Ay_C^2$$
$$82.33 = I_{x'} + 7(3.1)^2$$
$$I_{x'} = \ldots.$$

$$I_y = I_{y'} + Ax_C^2$$
$$2.58 = I_{y'} + 7(0)^2$$
$$I_{y'} = \ldots.$$

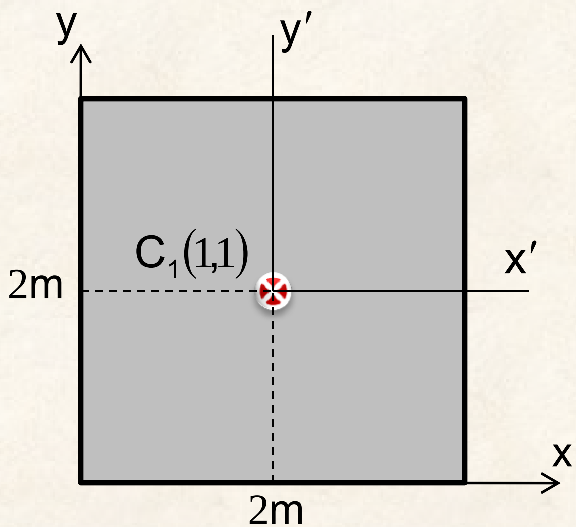
$$I_{xy} = I_{x'y'} + Ax_c y_c$$
$$0 = I_{x'y'} + 7(0)(3.1)$$
$$I_{x'y'} = 0$$

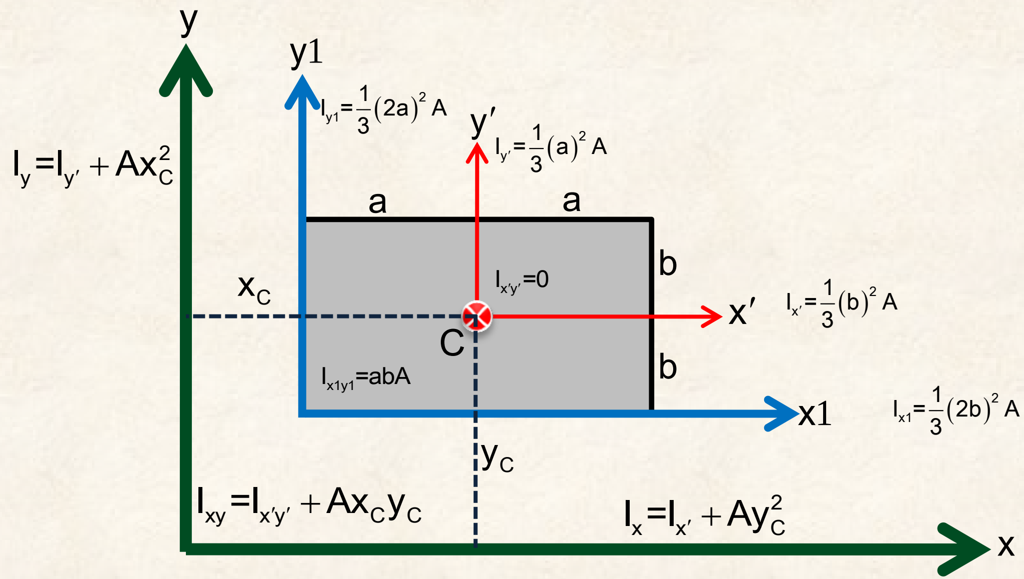
$$I_x = 82.33\,m^4$$
$$I_y = 2.58\,m^4$$
$$I_{xy} = 0$$
$$A = A_1 + A_2 = 7m^2$$

(cross section c)



(cross section c)





$$A_1 = 2x2 = 4m^2$$

$$I_{x1}=\frac{1}{3}(2b)^2 A=\frac{1}{3}(2)^2(4)=5.33m^4$$

$$I_{y1}=\frac{1}{3}(2a)^2 A=\frac{1}{3}(2)^2(4)=5.33m^4$$

$$I_{x1y1}=abA=(1)(1)(4)=4m^4$$

Remember

Given data

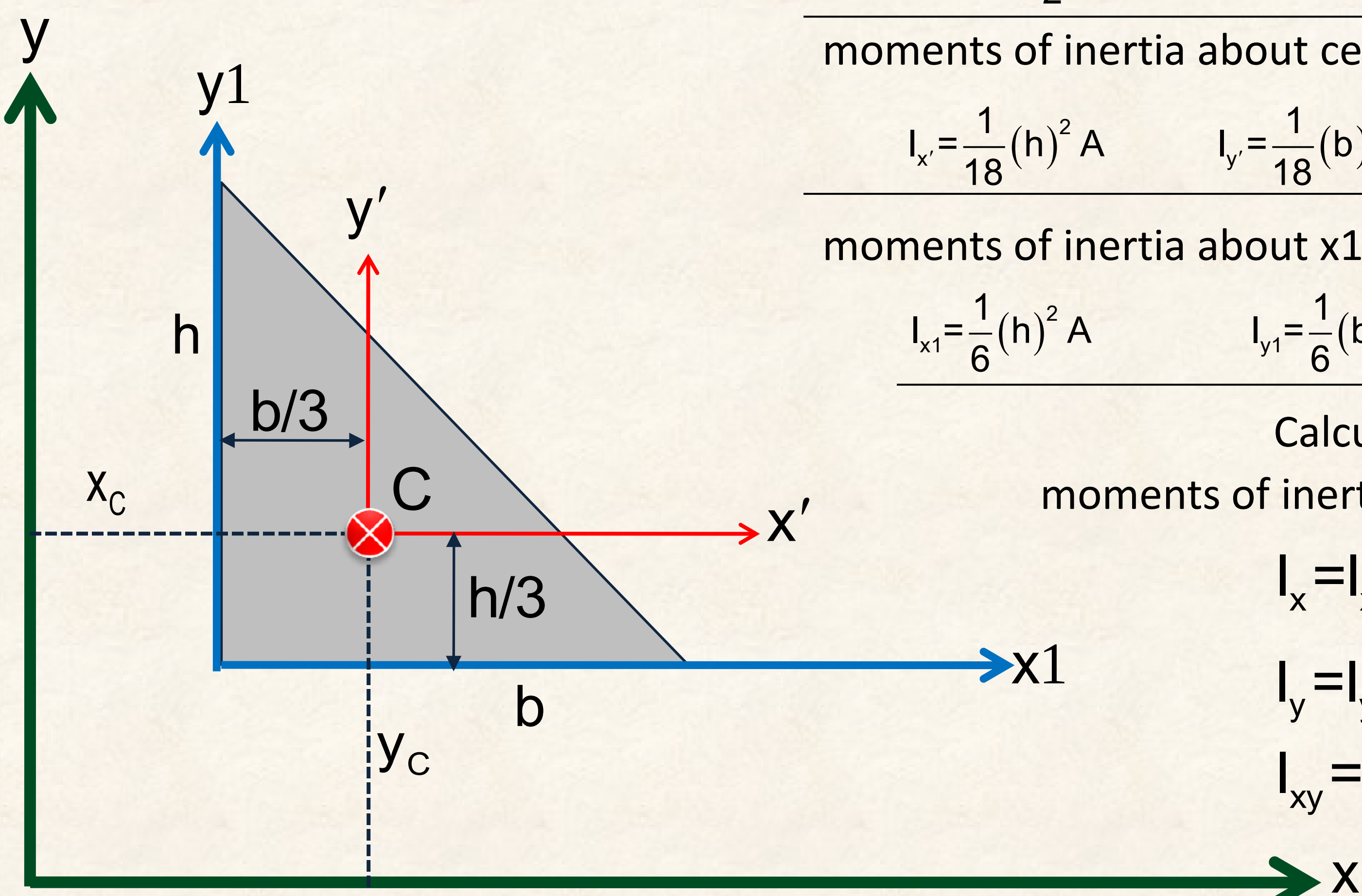
area $A=\frac{1}{2}hb$

moments of inertia about centroidal axes

$I_{x'}=\frac{1}{18}(h)^2A \qquad I_{y'}=\frac{1}{18}(b)^2A \qquad I_{x'y'}=\frac{-1}{36}hbA$

moments of inertia about x1 and y1 axes

$I_{x1}=\frac{1}{6}(h)^2A \qquad I_{y1}=\frac{1}{6}(b)^2A \qquad I_{x1y1}=\frac{1}{18}hbA$



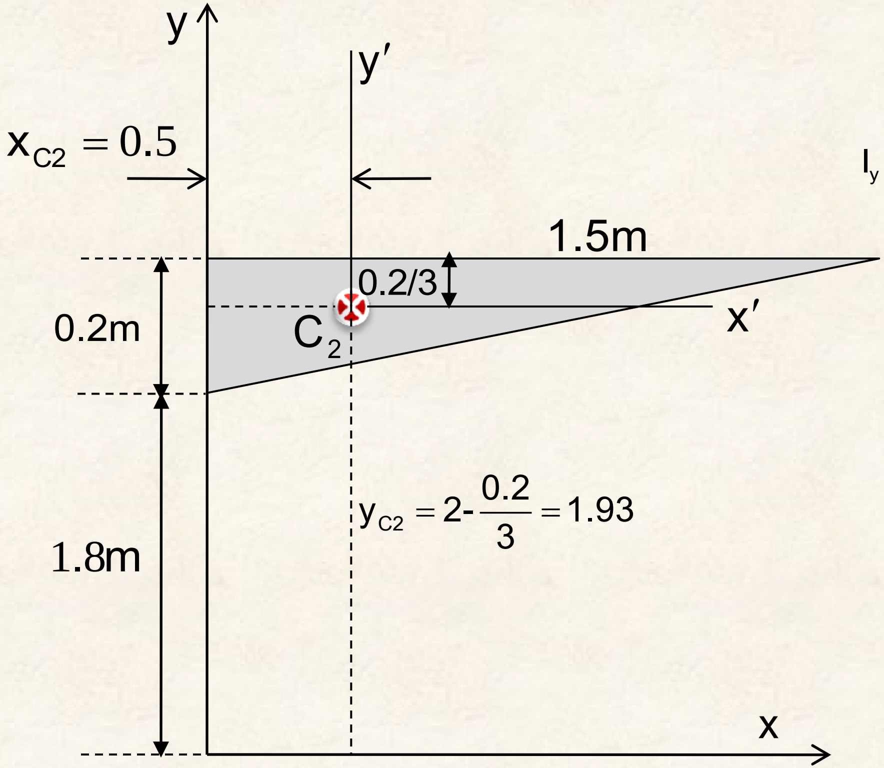
Calculated data

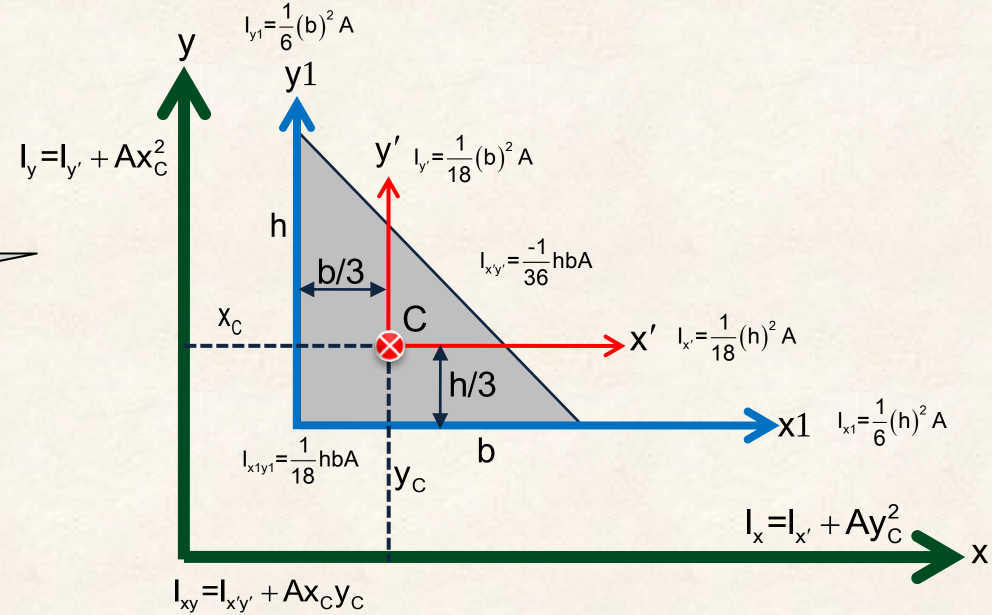
moments of inertia about x and y axes

$$I_x=I_{x'}+Ay_C^2$$

$$I_y=I_{y'}+Ax_C^2$$

$$I_{xy}=I_{x'y'}+Ax_Cy_C$$





$$A_2 = 0.2x1.5/2 = 0.15 m^2$$

$$I_{x2} = I_{x'2} + A_2 (y_{C2})^2$$

$$I_{x2} = \frac{1}{18}(0.2)^2(0.15) + 0.15(1.93)^2 = 0.56 m^4$$

$$I_{y2} = \frac{1}{6}(b)^2 A = \frac{1}{6}(1.5)^2(0.15) = 0.056 m^4$$

$$I_{xy2} = I_{x'y'2} + A_2 (x_{C2})(y_{c2})$$

$$I_{xy2} = \frac{1}{36}(1.5)(0.2)(0.15) + 0.15(0.5)(1.93) = 0.146 m^4$$

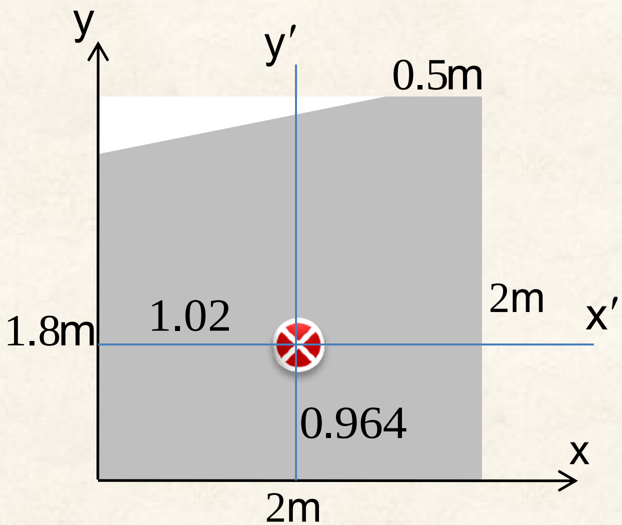
Total area moments of inertia

$$I_x = I_{x1} - I_{x2} = 4.8m^4$$

$$I_y = I_{y1} - I_{y2} = 5.3m^4$$

$$I_{xy} = I_{xy1} - I_{xy2} = 3.9m^4$$

moments and products of inertia about the centroidal axes



$$A_1 = 2x2 = 4m^2$$

$$A_2 = 0.2x1.5/2 = 0.15m^2$$

$$A = A_2 - A_1$$

$$I_x = I_{x'} + Ay_C^2$$

$$I_{x'} = ...$$

$$I_y = I_{y'} + Ax_C^2$$

$$I_{y'} = ...$$

$$I_{xy} = I_{x'y'} + Ax_c y_c$$